

EMF Power Absorption In Bone and Marrow: Mathematical Model

ABSTRACT

A deterministic mathematical method is adopted to evaluate the power absorption due to EMF radiation in bone and bone marrow. The specific absorption rate (SAR) in bone and bone marrow is computed and represented graphically to show spatial distribution in each. The mathematical model constructed is applied using the reported frequency dependent electromagnetic properties. The effect of exposure to electric field of strength ranging from 1V/m to 1kV/m is investigated, accordingly, for a wide frequency spectrum. The frequency dependence of the SAR through the bone-marrow-bone layers under study is illustrated for a frequency range of 1kHz-1GHz.

Aim: Evaluation of the power absorption and distribution, in bone and bone marrow, due to EMF radiation.

Study design: Mathematical analysis then computer simulation of the problem.

Place and Duration of Study: Department of Engineering Physics & Math., Faculty of Engineering, Cairo University, between May 2014 and Dec.2015.

Methodology: The author employs a bone-marrow-bone model to investigate the effect of incident EMF. The equations governing the total electric and magnetic field distributions in each layer is deduced, considering its biological electromagnetic properties. The model is simulated by a computer program using Maple V. The computed values of specific absorption rate (SAR) in bone and bone marrow are graphically represented to show spatial distribution in each. The exposure to electric field of strength ranging from 1V/m to 1kV/m is investigated using the proposed method. The frequency dependence of the SAR through the bone-marrow-bone layers under study is illustrated for a frequency range of 1kHz-1GHz.

Results: Electromagnetic radiation of 1MHz-10MHz induce absorbed power within the safety limits for all applied field strengths. The 1GHz incident radiation induces SAR values higher than permissible ranges for field strengths above 400V/m whereas the same occurs for the low frequency range at 100V/m. Moreover, the present results are in agreement with international safety standards for applied field strengths till 10V/m for bone and till 100V/m for marrow, covering the applied frequencies (1 kHz -1 GHz). Except for exposure to electric field of strength higher than 100V/m, the SAR acquired by the marrow is within the safety levels.

Conclusion: The present results are in agreement with international safety standards for field strengths of maximum value 10V/m for bone and 100V/m for marrow. Oblique incidence results in higher SAR values than normal incidence, especially for low frequency (1kHz).

Keywords: Specific absorption rate- Bone-Bone marrow-EMF radiation- Power absorption- Field strengths- Frequency dependence

15 1. INTRODUCTION

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17 Recently, the intense existence of electromagnetic environment accompanying the
18 progressive applications of electromagnetic fields; have represented a growing threat to the
19 public health. Various electronic devices that employ EMF such as, cellular phones and their
20 networks, microwave transmitters, antennae, etc. impose significant biological effects.
21 Hence, investigating the EMF radiations interaction with tissues and assessing their effect on
22 biological systems have occupied a considerable scientific attention [1-5].

23 There are three major physical quantities to be determined for evaluating the effect of EMF
24 exposure; namely the flowing current per unit length through the body; the energy density
25 the tissues might be subjected to and the amount of power absorbed per unit mass of
26 biological tissue. The specific absorption rate, SAR, evaluation is the most commonly
27 accepted quantity for international standardization [6-11]. The study of the possible hazards
28 of the EM exposure is either experimentally performed on animals [12, 13] or assessed by
29 mathematical approaches that employ either a defined set of mathematical equations or
30 stochastic modelling [14-18]. Furthermore, using computer simulators, employing either
31 frequency or time domain analysis, is a dependable method for SAR measurements. In
32 2008, D. Smith [19] has produced an extensive report concerning the EM field propagation
33 loss through different human body phantom sections, using SEMCAD X. Antennae
34 employed are directive and positioned near the body. Finite difference frequency domain,
35 FDTD, is another method that has been employed to represent the EMF distribution through
36 a human head phantom [20]. EMF has been emitted from mobile antenna placed at different
37 distances from the head.

38 However, the adverse effect of the EM fields remains a potent source of controversy.
39 There is no sufficient, reliable evidence to confirm or deny whether long-term exposure to
40 these fields have an adverse health effect.

41 Mathematical modelling, adopted in the present work, is deterministic depending on tracing
42 the wave propagation through a multilayer section of bone-marrow-bone. Maxwell equations
43 are employed together with the physical and electromagnetic properties of the biological
44 tissues under consideration, to study their EMF interaction. An electromagnetic wave is thus
45 assumed to be incident on a homogeneous multilayer bone-marrow-bone section. A
46 mathematical simulation model is thus applied to calculate the root mean square value of the
47 electric and magnetic fields, hence the electromagnetic power density absorbed in each
48 layer. Bone and marrow are considered as non-magnetic materials, hence their magnetic
49 permeability is less effective than their permittivity and conductivity are. Skin effect is
50 neglected as it is only significant at VHF ranges. Hence, the power absorption is studied as a
51 function of both, the frequency (1 kHz-1 GHz) and the electric field strength (1 V/m-1 kV/m).
52 The main goal of the present work is to introduce an approach to the problem of calculating
53 the average power absorbed by bone marrow and hence compare the results to the
54 approved safety standards. Computations are performed using Maple-V software. The
55 program is constructed by the author to compute the total electric and magnetic fields and
56 their root mean squared values in respective layers.

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58 2. MATHEMATICAL METHODOLOGY

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60 In the present work, the problem of electromagnetic wave incidence on a dissipative
61 medium, namely biological tissue is investigated. Firstly, two planar sections of successive
62 bone with marrow in between is assumed to be subjected to incident polarized
63 electromagnetic wave, in the far field. Incident electromagnetic energy is transmitted through
64 bone to marrow layer. The reflection on successive interfaces contributes to the overall
65 energy consumed in each layer. Fundamental constants defining the reflected and
66 transmitted fields are the electrical and magnetic parameters of the medium, permittivity, $\epsilon(f)$,

67 conductivity, $\sigma(f)$, and permeability, $\mu(f)$ for each layer. $k(f)$ is the wavenumber for each
 68 layer. The incident polarized electric field is assumed to be propagating in the x-direction,
 69 represented by, $E_i(t,x)$ and $H_i(t,x)$ as thus:

$$70 \quad E_i(x, t) = E_0 \times e^{i(2\pi ft - k_0 x)} \quad (1)$$

$$71 \quad H_i(x, t) = \sqrt{\mu_1(f)\epsilon_1(f)} E_0 \times e^{i(2\pi ft - k_0 x + \pi/2)} \quad (2)$$

72 The posterior bone layer is denoted as layer 1, the marrow as layer 2, and the anterior bone
 73 layer as 3. Mathematical analysis is adopted to calculate the electric and magnetic field
 74 distributions in the three consecutive layers. To avoid complexity and redundancy of
 75 equations, the transmitted and reflected horizontal components of electromagnetic field
 76 through the marrow are given below:

$$77 \quad E_{t2}(x, t) = t_{h1}(f) \times E_0 \times e^{-\delta_1(f)\frac{\Delta x_1}{2} + i(2\pi ft - k_2 x)} \quad (3)$$

$$78 \quad E_{r2}(x, t) = t_{h1}(f) r_{h2}(f) \times E_0 e^{-\delta_1(f)\frac{\Delta x_1}{2} - \delta_2(f)\frac{\Delta x_2}{2} + i(2\pi ft + k_2 x)} \quad (4)$$

$$79 \quad H_{t2}(x, t) = t_{h1}(f) \sqrt{\mu_2(f)\epsilon_2(f)} \times E_0 \times e^{-\delta_1(f)\frac{\Delta x_1}{2} + i(2\pi ft - k_2 x + \pi/2)} \quad (5)$$

$$80 \quad H_{r2}(x, t) = t_{h1}(f) r_{h2}(f) \sqrt{\mu_2(f)\epsilon_2(f)} \times E_0 \times e^{-\delta_1(f)\frac{\Delta x_1}{2} - \delta_2(f)\frac{\Delta x_2}{2} + i(2\pi ft + k_2 x + \pi/2)} \quad (6)$$

81 $t_{h1}(f)$, $r_{h2}(f)$, and $\delta_2(f)$ are transmission, reflection and absorption coefficients of the medium.
 82 Similarly, the vertical components of the field can be determined using the vertical reflection
 83 and transmission coefficients, Eq.(8).

$$84 \quad r_{h2}(f) = \frac{\sqrt{\epsilon_3} / \sqrt{\epsilon_2} - \cos\theta_3 / \cos\theta_2}{\sqrt{\epsilon_3} / \sqrt{\epsilon_2} + \cos\theta_3 / \cos\theta_2} \quad t_{h1}(f) = \frac{2}{\sqrt{\epsilon_2} / \sqrt{\epsilon_1} + \cos\theta_2 / \cos\theta_1} \quad (7)$$

$$85 \quad r_{v2}(f) = \frac{1 - \sqrt{\epsilon_3} \cos\theta_3 / \sqrt{\epsilon_2} \cos\theta_2}{1 + \sqrt{\epsilon_3} \cos\theta_3 / \sqrt{\epsilon_2} \cos\theta_2} \quad t_{v1}(f) = \frac{2}{1 + \sqrt{\epsilon_2} \cos\theta_2 / \sqrt{\epsilon_1} \cos\theta_1} \quad (8)$$

86 Considering that the present study aims at the assessment of power absorption,
 87 electromagnetic power density vector, $S_{tot}(t,x)$, in a specific layer is represented as:

$$88 \quad S_{tot}(x, t) = E_{tot}(x, t) \times H_{tot}(x, t) \quad (9)$$

89 where $E_{tot}(x, t)$ and $H_{tot}(x, t)$ are the total electric and magnetic fields in the respective
 90 layer.

91 The mathematical derivation, introduced in the present work, produces the total electric field,
 92 $E_{tot}(x, t)$, in the marrow layer, of thickness Δx_2 as:

$$93 \quad E_{tot}(x, t) = t_{p1} t_{p2} e^{-\delta_1 \frac{\Delta x_1}{2}} E_0 \left\{ \left(1 - e^{-\delta_2 \frac{\Delta x_2}{2}} r_{p2} \cos(k_1 \Delta x_1 + 2k_2 \Delta x_2) \right) \sin(2\pi ft - kx) - \right. \\ \left. e^{-\delta_2 \frac{\Delta x_2}{2}} r_{p2} \sin(k_1 \Delta x_1 + 2k_2 \Delta x_2) \cos(2\pi ft - kx) \right\} \quad (10)$$

94 Δx_1 and Δx_3 denote the posterior and anterior bone thicknesses respectively. Hence, the
 95 root mean square value of $E_{tot}(x, t)$, $E_{rms}(x)$, is deduced from Eq.10 giving:

$$96 \quad E_{rms}(x, f) = \left\{ \frac{t_{p1}^2 t_{p2}^2 E_0^2}{8\pi} (1 - r_{p2} \cos(k_1 \Delta x_1 + 2k_2 \Delta x_2))^2 \right. \\ \left. + \frac{1}{8\pi} r_{p2}^2 \sin^2(k_1 \Delta x_1 + 2k_2 \Delta x_2) \right\} \{ \sin(4\pi f + 2k_2 x) + \sin(2k_2 x) + \frac{1}{2} \} \quad (11)$$

97 Similar derivations are carried out for the posterior and anterior bone layers. The incident
 98 field on a specific layer is that transmitted from the previous one. Reflection and
 99 transmission occurs at each interface. Similarly, $H_{rms}(x)$ and $H_{tot}(x, t)$ are deduced,
 100 hence $S_{tot}(x, t)$ absorbed in each layer can be calculated.

101 The specific absorption rate, SAR, being dependent on the electric field root mean squared
 102 value, E_{rms} , is averaged over any thickness Δx as:

$$103 \quad SAR(f) = \frac{1}{\Delta x} \int_0^{\Delta x} \frac{\sigma(f)}{\rho} E_{rms}^2(x, f) dx \quad (12)$$

104 The frequency dependence of the SAR function is thus complicated, considering the
 105 frequency dependence the electromagnetic properties involved.

106 **3. RESULTS**

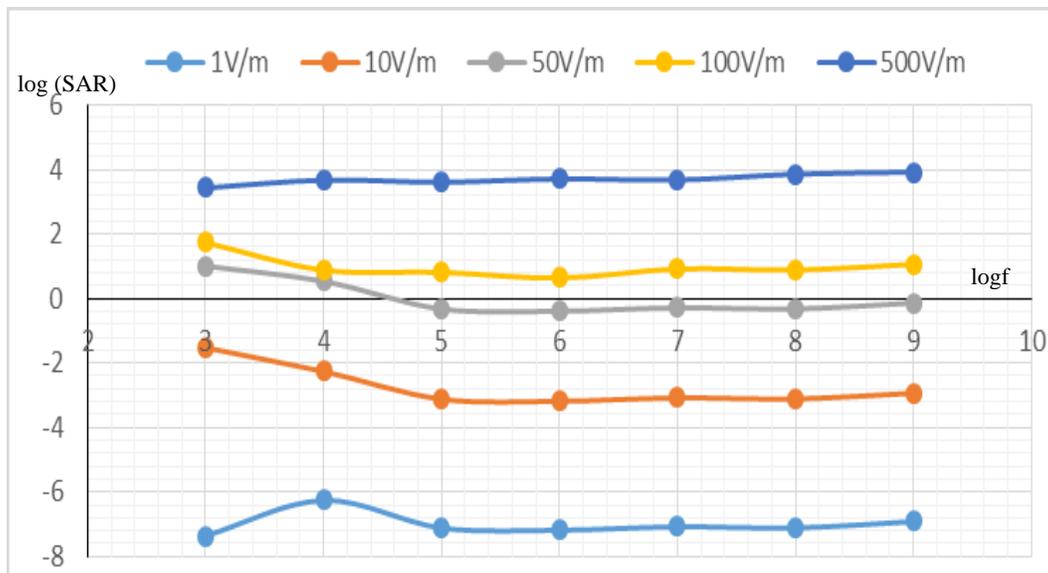
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108 A double layer of bone section, 3mm thickness each, with a 5mm marrow layer in between,
 109 is subjected to incident electromagnetic waves. Horizontally polarized incident fields are
 110 assumed to be incident on a unit area of the section. Considerable biological tissues are
 111 assumed to be homogeneous. The root mean square of the phasor addition of the
 112 transmitted and the reflected electric fields is calculated for each layer as in Eq.11. The
 113 mathematical model is applied to illustrate the SAR variation with frequency. In addition to
 114 this, the relation between the SAR and the incident electric field strength, E_0 , for a wide
 115 frequency spectrum is represented. The spatial distribution of the SAR function, through the
 116 successive layers, is then calculated and represented as well. The electromagnetic
 117 parameters are actual reported data for cortical bone and marrow. The model applied
 118 depends greatly on the frequency dependent media parameters reported by references [21-
 119 24]. A horizontally polarized plane wave is assumed to be normally incident on a 1mm^2
 120 surface of the bone-marrow-bone layers. The SAR function, due to the horizontally polarized
 121 electric field with normal incidence, is calculated. Figures (1-a,1-b) illustrate the rise of SAR
 122 function, in log scale, versus the frequency in log scale as well, for different electric field
 123 strengths for both bone and marrow layers. Figures (2-a,2-b) represent the spatial
 124 distribution of the SAR function across the bone- marrow-bone section. These figures show
 125 the change of pattern for 1kHz and 1GHz.
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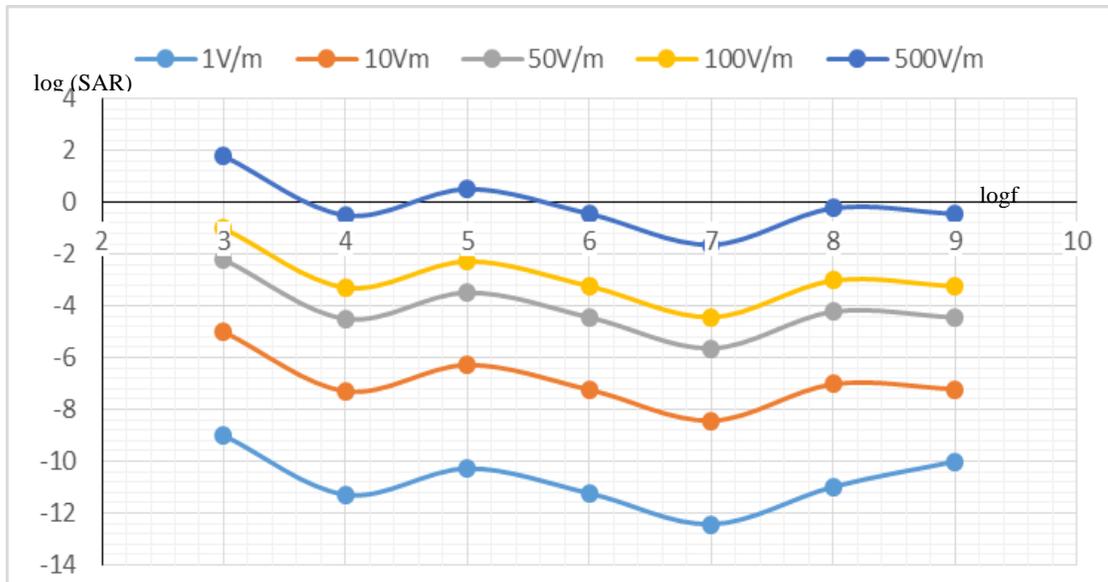
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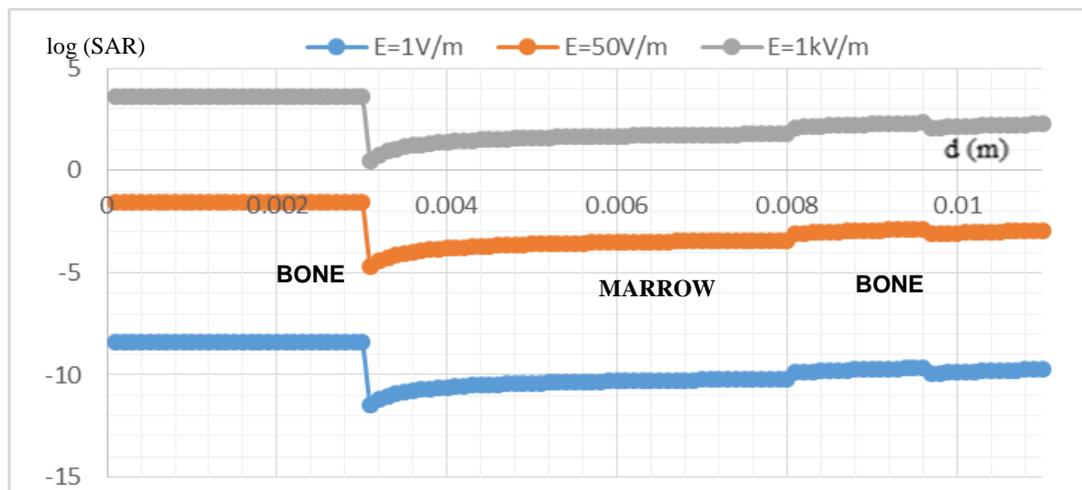
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Fig.1-a log(SAR) vs log(f) for bone



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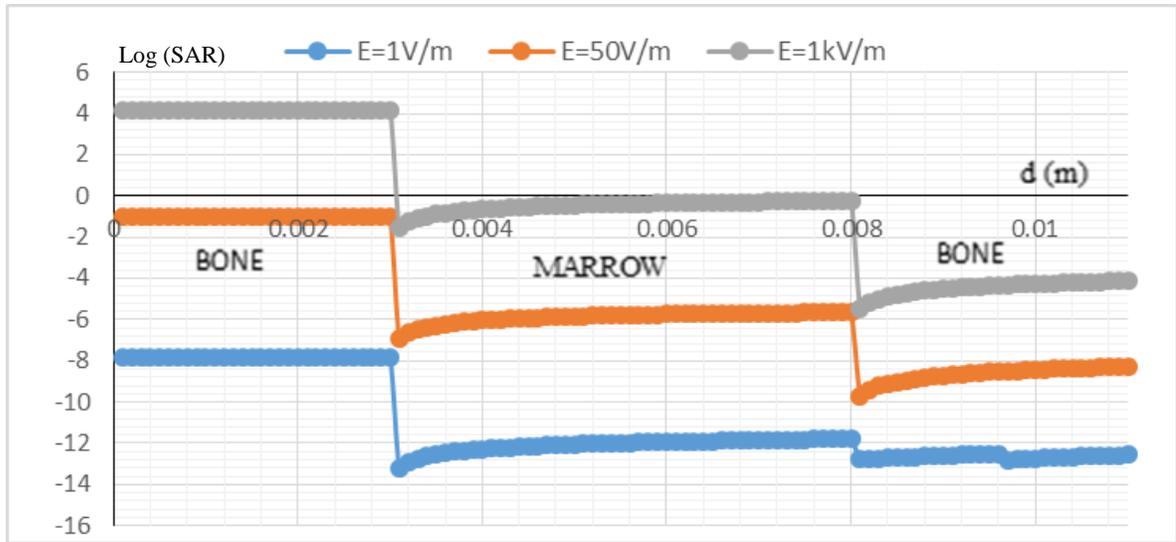
Fig.1-b log(SAR) vs log(f) for marrow



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Fig.2-a log(SAR) vs depth (m) for bone-marrow-bone layers calculated at 1kHz and different values of vertically polarized electric field strengths.

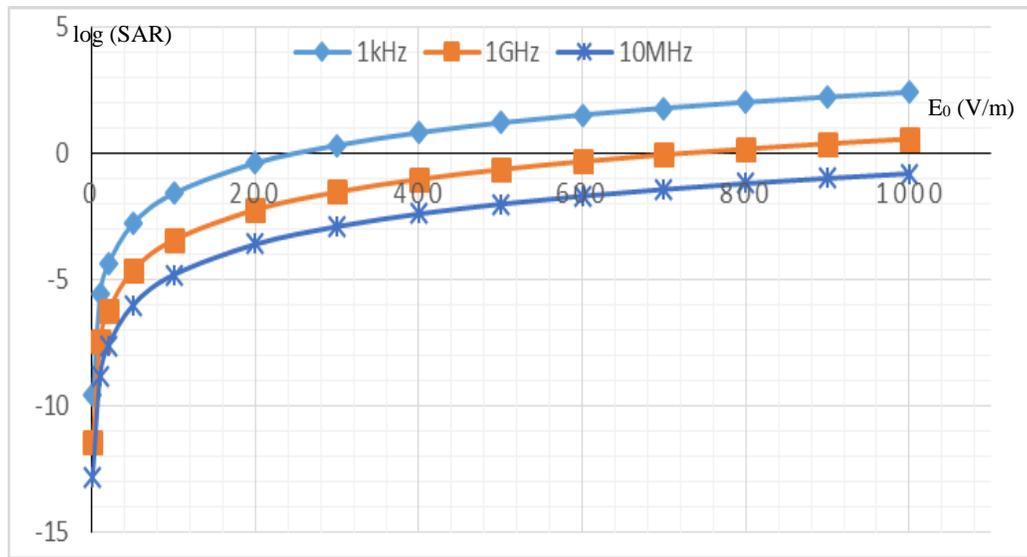
136 The bone thickness, in the range (0.1 mm- 5 mm), does not affect the SAR value. Figure (3)
137 illustrates the rise of SAR function, in log scale, versus the incident electric field strength, E_0
138 in V/m, plotted at 1 kHz, 10 MHz and 1 GHz.
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Fig.2-b $\log(\text{SAR})$ vs depth (m) for bone-marrow-bone layers calculated at 1GHz and different values of vertically polarized electric field strengths

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Fig.3 $\log(\text{SAR})$ vs E_0 (V/m) calculated for different frequencies

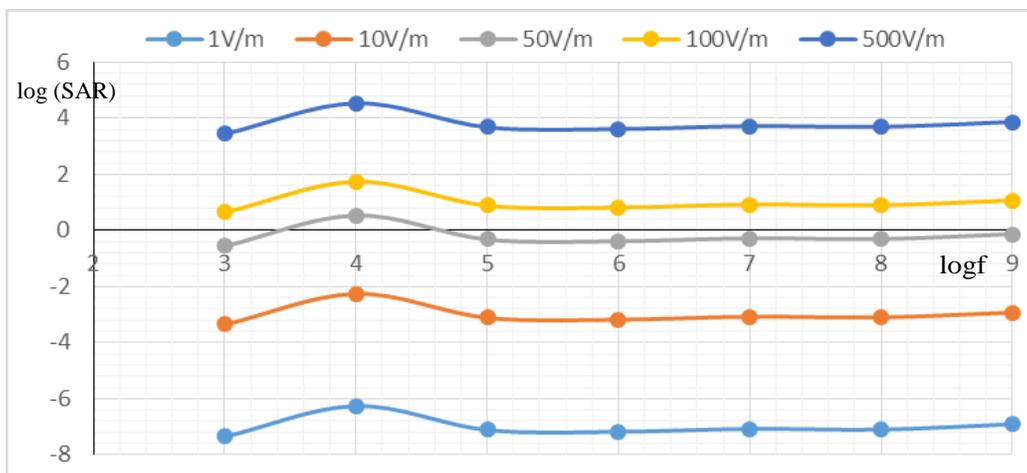


Fig.4-a log(SAR) vs log(f) for bone calculated for different values of electric field strengths with oblique incidence $\theta=30^\circ$

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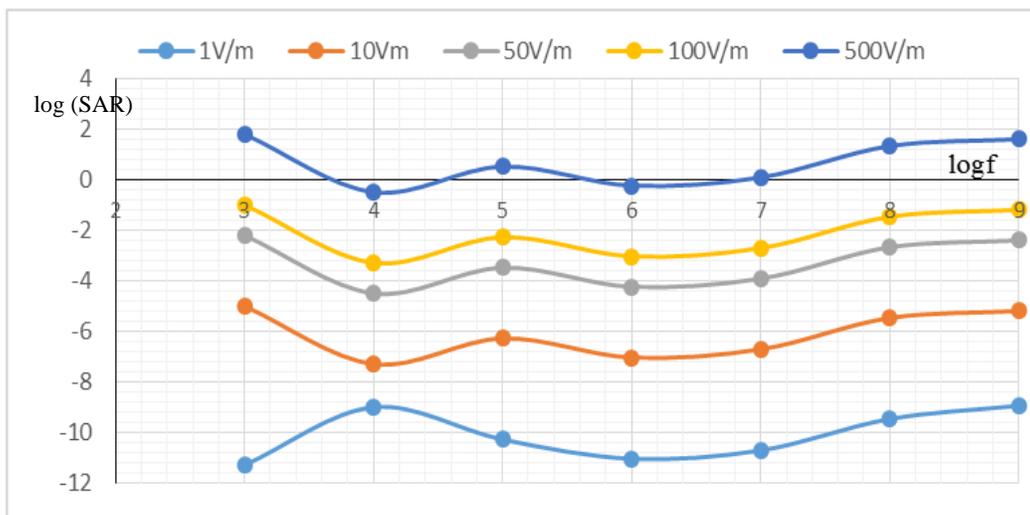


Fig.4-b log(SAR) vs log(f) for bone marrow calculated for different values of electric field strengths with oblique incidence $\theta=30^\circ$

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For oblique incidence, figures (4-a,4-b) illustrate the rise of SAR function, in log scale, versus the frequency in log scale as well, for different electric field strengths for both bone and marrow layers. The SAR values are calculated due to horizontally polarized electric field incident at an angle of incidence 30° .

4. DISCUSSION AND CONCLUSION

Electromagnetic interactions with biological tissue are a potent source of controversy. Not only the possible health effects is the controversial but also the mechanism that leads to these effects is under constant debate. It is not well established whether this effect is thermal, caused by high frequency vibrations of the molecules, or non-thermal that could cause serious disturbance on the cell membrane or even the DNA. [Zhong et al. \[12\]](#) reported

167 the harmful effects of low intensity electromagnetic field (0.5 mT, 50 Hz), on bone marrow,
168 increasing cell proliferation and inducing cell differentiation. While, Prisco et al. [13]
169 investigated the effects of GSM-modulated radiofrequency electromagnetic waves on bone
170 marrow.

171 However, scientific communities periodically issue safety standards concerning EMF
172 exposure. FCC recommend that SAR international standards, for mobile phones and their
173 networks, not to exceed 0.04 W/kg [25]. Harmonization of ICNIRP and IEEE has been
174 established between their standard limits. Their latest reports have restricted the safe SAR
175 limits of the whole-body exposure to 0.4W/kg and the partial body exposure to 10 W/kg for
176 occupational exposure. For public exposure, SAR limit for the whole body is 0.08 W/kg and
177 for the partial body is 2W/kg [9, 10, 26, 27]. European standards limit the maximum public
178 exposure level to 1.6W/kg [7].

179 The present work proposes a methodology based on mathematical formulation of EMF
180 penetration through bone. It complements the SAR values resulting from other phantom and
181 mathematical modeling [12-13]. This methodology is suitable for studying other complicated
182 tissues, however the author was interested in obtaining the SAR values absorbed by bone
183 and bone marrow.

184 The present results show that electromagnetic radiation of 1MHz-10MHz are within the
185 safety limits for all applied field strengths. These results also show that the 1GHz frequency
186 radiation show SAR values higher than permissible ranges for field strengths above 400V/m
187 whereas the same occurs for the low frequency range at 100V/m. Moreover, the present
188 results are in agreement with international safety standards for applied field strengths till
189 10V/m for bone and till 100V/m for marrow, covering the applied frequencies (1 kHz -1 GHz).
190 Except for exposure to electric field of strength higher than 100V/m, the SAR acquired by the
191 marrow is within the safety levels. Furthermore when oblique incidence is applied the SAR
192 values are higher than with normal incidence case, especially for low frequency (1kHz).

193 On the other hand, some limitations of the present method ought to be mentioned; firstly
194 the direction of propagation being taken very specific while in real cases the field is spatially
195 random. Moreover, the present approach is only applicable to far field exposure. This is the
196 common case for public exposure to different sources of radiation. Secondly, not only the
197 reported physical properties of bone and marrow are very scarce, but their actual
198 dimensions differ considerably with sex, age and state of health. Despite the fact that at low
199 frequency range, international standardization takes into account the current density instead
200 of SAR, the results presented in this paper extend the SAR calculation to any frequencies.
201 The present work has no previous parallel as most of the researchers did not examine
202 experimentally the absorption of EMF due to the extreme difficulties to perform non -
203 destructive tests *in vivo* or even *in vitro*. Moreover, numerical methods, employed using
204 computer simulators to analyse EMF interaction with human body phantom, are usually
205 investigating fields due to antennae either placed close to or implanted inside it. Hence
206 producing data applicable for directive near field regions.

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