The thermodynamics of a gravitating vacuum

Review Paper

Abstract

In the present days of modern cosmology it is assumed that the main ingredient to cosmic energy presently is vacuum energy with an energy density ϵ_{vac} that is constant over the cosmic evolution. In this paper here we show, however, that this assumption of constant vacuum energy density is unphysical, since it conflicts with the requirements of cosmic thermodynamics. We start from the total vacuum energy including the negatively valued gravitational binding energy and show that cosmic thermodynamics then requires that the cosmic vacuum energy density can only vary with cosmic scale $R = R(t)$ according to $\epsilon_{\mathrm{vac}} \sim R^{-\nu}$ with only two values of ν being allowed, namely $\nu_1 = 2$ and $\nu_2 = 5/2$. We then discuss these two remaining solutions and find, when requiring a universe with a constant total energy, that the only allowed power index is $\nu_1 = 2$. We discuss the consequences of this scaling of ϵ_{vac} and show the results for a cosmic scale evolution of a quasi-empty universe like the one that we are presently faced by.

Keywords: Cosmic vacuum energy density; Friedmann equations; Thermodynamics

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1 Introduction

We start this paper asking why at all should a vacuum gravitate or influence spacetime geometry? This question is perhaps worth to be asked, since, if vacuum, expressis verbis, represents 'nothing' in a physical sense, then it should not do anything, especially should not gravitate, unless it is wrongly defined. Modern physics nowadays argues, however, that a vacuum cannot be energy-less, but is loaded with energy, or, due to the energy-mass equivalence, is mass-loaded. Masses, on the other hand, do in general gravitate, unless something else compensates for that. But how could sources of gravity be compensated, unless perhaps by anti-masses which are not known to exist?

The General Relativistic action of a vacuum in general is taken into account by a fluid-like hydrodynamical energy-momentum tensor $T_{\mu\nu}^{vac}$ which describes how the vacuum, due to its pressure p_{vac} and its mass energy density ρ_{vac}, acts as source of spacetime geometry (see e.g. [Goenner (1996)]). If in addition vacuum energy density $\epsilon_{vac} = \rho_{vac}c^2$ is assumed to be constant, as done in present-day standard cosmologies (see [Perlmutter et al. (1999)]; [Bennett (2003)]), then this induces the relation $p_{vac} = -\epsilon_{vac}$ (see e.g [Peebles and Ratra (2003)] and leads to the following geometrical source tensor (see e.g. [Overduin and Fahr (2003)]) $T_{\mu\nu}^{vac} = \rho_{vac}c^2 g_{\mu\nu}$, where $g_{\mu\nu}$ denotes the metric tensor.

This term $T_{\mu\nu}^{vac}$, since being isomorphal, can be taken together with the term due to Einstein's cosmological constant Λ_0 ([Einstein (1917)]). If both terms are placed on the right-hand side of the GRT field equations, while Einstein placed his term on the left hand side, they can be put together representing an 'effective' cosmological constant Λ_{eff} given by [Overduin and Fahr (2001)] and [Fahr (2004)].

$$\Lambda_{eff} = \frac{8\pi G}{c^2}\rho_{vac,0} - \Lambda_0. \tag{1.1}$$

Now one can draw the following conclusion: A completely empty, matter-free space, not doing anything in terms of gravity, is realized, if ,evident from the above, Λ_{eff} just vanishes, i.e. the cosmological term Λ_0 just compensates the vacuum energy density of empty space whatever maybe its value (e.g. see [Zeldovich (1968)] and [Carrol (1992)]).

Interestingly, very similar ideas have come up in papers by Sola (see [Solà (2013)] and [Solà (2014)]) who expresses the fact that in order to settle down the spacetime geometry of a pure vacuum to a nongravitating Minkowskian spacetime within a covariant general-relativistic field theory the effective vacuum energy of this empty space has to vanish.

In the presence of real matter the argumentation, however, is much more complicate as we have discussed at several places in the literature ([Overduin and Fahr (2001)]; [Fahr (2004)]; [Fahr and Heyl (2007a)]; [Fahr and Heyl (2007b)]; [Fahr and Sokaliwska (2012)]). Especially it is then highly questionable whether under such conditions a constant vacuum energy density can at all be expected as an option.

If under these perspectives it could be assumed, that only the energy difference between the matter-polarized and the empty vacuum gravitates then some interesting new conclusions could be drawn. It then means that in a matter-filled universe the effective quantity representing the action of the vacuum energy density is given by:

$$\Lambda_{eff} = \frac{8\pi G}{c^2}(\rho_{vac} - \rho_{vac,0}). \tag{1.2}$$

The above formulation expresses that in a matter-filled universe only the difference between the values of the vacuum energy densities $\rho_{vac,0}$ of empty space and ρ_{vac} of matter-polarized space gravitates, i.e. the spacetime geometry only reacts to the difference of these vacuum energies.

Even under these new prerequisites it is nevertheless not the most natural assumption, that vacuum energy density $\epsilon_{vac} = \rho_{vac}c^2$ should be considered as a time-independent quantity. This is because the unit of volume is not a cosmologically relevant quantity, and vacuum energy density neither is. It would probably appear more reasonable to assume that the energy load of any homologously comoving proper volume does not change with cosmic expansion, i.e. that rather just this proper-energy is constant. This demand, however, means that the true constant quantity, instead of the vacuum energy density ϵ_{vac}, is

$$e_{vac} = \epsilon_{vac}\sqrt{-g_3}d^3V \tag{1.3}$$

where g_3 is the determinant of the 3d-space metric which in case of a Robertson-Walker geometry is given by

$$g_3 = g_{11}g_{22}g_{33} = -\frac{1}{(1-Kr^2)}R^6 r^4 \sin^2\vartheta \tag{1.4}$$

with K denoting the curvature parameter, the $R = R(t)$ determining the time-dependent scale of the universe, and the differential 3-space volume element in normalized polar coordinates given by

$$d^3V = drd\vartheta d\varphi. \tag{1.5}$$

This then leads to the following request

$$\begin{aligned} e_{vac} = \epsilon_{vac}\sqrt{R^6 r^4 \sin^2\vartheta/(1-Kr^2)}drd\vartheta d\varphi = \\ \epsilon_{vac}\frac{R^3}{\sqrt{1-Kr^2}}r^2 \sin\vartheta drd\vartheta d\varphi = const. \end{aligned} \tag{1.6}$$

which evidently leads to a variability of the vacuum energy density ϵ_{vac}in the form

$$\epsilon_{vac} = \rho_{vac}c^2 \sim R(t)^{-3}. \tag{1.7}$$

In the following paper we shall now throw some new light on the variability of ϵ_{vac} that must be expected. We therefore study the behavior of the vacuum energy density ϵ_{vac} with the scale $R(t)$ of the universe from a thermodynamical view.

2 Thermodynamics of the cosmic vacuum

In the following cosmological considerations we treat the cosmic vacuum by quantities denoting its vacuum energy density ε_{vac} and its associated vacuum pressure p_{vac}, like done in case of a hydrodynamic fluid which in general relativity theory is described by the following fluid-type hydrodynamical energy-momentum tensor (see e.g [Goenner (1996)]; [Overduin and Fahr (2001)]; [Blome (2002)]; [Fahr (2004)])

$$T^{vac}_{\mu\nu} = (\rho_{vac}c^2 + p_{vac})U_\mu U_\nu - p_{vac} * g_{\mu\nu} \tag{2.1}$$

where $\varepsilon_{vac} = \rho_{vac}c^2$ and p_{vac} are energy density and pressure of the vacuum, U_i denote the components of the fluid four-velocity, and $g_{\mu\nu}$ is the four-space metric tensor.

In order to use the above energy-momentum tensor in the frame of the general relativistic field equations one needs to know, how ρ_{vac} and p_{vac} are related to each other and how they are dependent on spacetime coordinates. For that purpose we want to use the well known thermodynamic equation that relates the internal volume energy with the work expended at the expansion of that volume. In its easiest form for a Robertson-Walker symmetric universe with curvature $K = 0$ this equation for a sphere of scale $R = R(t)$ is given by (see [Goenner (1996)]):

$$\frac{4\pi}{3}\frac{d}{dR}(\varepsilon_{vac}R^3) = -p_{vac}\frac{4\pi}{3}\frac{d}{dR}R^3. \tag{2.2}$$

Analogously to a star at its contraction the internal volume energy, irrelevant whether it is vacuum- or matter-filled, should, however, be completed by the gravitational self-binding energy, since a vacuum that is energy-loaded evidently is a source of internal gravity which at all makes it cosmologically relevant as source of cosmic geometry. If we include the negatively valued gravitational self-binding energy (see [Fahr and Heyl (2007a)]; [Fahr and Heyl (2007b)]) into the total internal energy of a cosmic sphere with radius R, then instead of the above relation one obtains the following more complicate thermodynamic equation:

$$\frac{d}{dR}[\frac{4\pi}{3}\varepsilon_{vac}R^3 - \frac{8\pi^2 G}{15c^4}(\varepsilon_{vac} + 3p_{vac})^2 R^5] =$$

$$-p_{vac}\frac{4\pi}{3}\frac{d}{dR}R^3 \tag{2.3}$$

which now instead of Eq. (2.2) should define the relation between ε_{vac} and p_{vac} and both their dependences on the scale parameter $R = R(t)$ which is a function of the cosmic time t.

As evident, in this highly symmetric FLRW universe both quantities, i.e. ε_{vac} and p_{vac}, can only depend on the scale parameter $R(t)$. We now try to solve the above equation, following the same way as already used in the case of the more simple, uppermost thermodynamic Eq. (2.2), namely assuming a power-law dependence of ε_{vac} on R in the form $\varepsilon_{vac} \sim R^{-\nu}$ with an undefined power index ν, and then obtaining for the vacuum pressure the relation

$$p_{vac} = -\frac{3-\nu}{3}\varepsilon_{vac}. \tag{2.4}$$

Here so far all power indices, especially the cardinal index values $\nu = 0, 1, 2, 3,$ were equally allowed, none of them being apriori excluded, however, the R-dependence of p_{vac} and ε_{vac} turned out to be identical.

If we now make use of these earlier results (Eq. (2.4), but try to find solutions of the extended thermodynamic Eq. (2.3) on the basis of these earlier findings we then obtain:

$$\begin{aligned}-\frac{4\pi}{3}\frac{3(3-\nu)}{3-\nu}p_{vac}R^2 = -3\frac{4\pi}{3}p_{vac}R^2 + \\ \frac{8\pi^2 G}{15c^4}\frac{d}{dR}[(\varepsilon_{vac}+3p_{vac})^2R^5]\end{aligned} \tag{2.5}$$

which, since the terms left and right of the identity sign cancel, after replacing ε_{vac} by $p_{vac}$with Eq. (2.4) leads to the requirement

$$0 = \frac{(6-3\nu)^2}{(3-\nu)^2}\frac{d}{dR}(p_{vac}^2R^5). \tag{2.6}$$

This equation for a completed thermodynamics now evidently is only solved by two special values of ν, i.e. the requirements:

a: $\nu = \nu_1 = 2$

and

b: $p_{vac}^2R^5 = const$, i.e. by $\nu = \nu_2 = 5/2$

thus now determining, compared to the earlier result, a much more restricted set of physically possible dependences of p_{vac} and ε_{vac} on R.

3 Do there exist two competing solutions?

From the above derivation the two solutions $\nu = \nu_1$ and $\nu = \nu_2$ are competing as equally justified, and one could think of taking a representation of the form

$$\varepsilon_{vac} = \varepsilon_{0,1}(R/R_0)^{-\nu_1} + \varepsilon_{0,2}(R/R_0)^{-\nu_2} \tag{3.1}$$

as the most general solution. However, without any concrete, specific physics behind the different forms, how ε_{vac} reacts to cosmic scale expansion, this form of a solution is not really satisfying. Thus we try to restrict the possible power indices even more by looking at this question from another view.

Requiring a universe where in every instant the positively valued vacuum energy is compensated by its gravitationally induced self-binding energy, then , in addition to the above thermodynamic requirement, one has to also fullfill the following relation (see [Fahr and Heyl (2007a)]; [Fahr and Heyl (2007b)]) for a vanishing total vacuum energy

$$\frac{4\pi}{3}(\varepsilon_{vac} + 3p_{vac})R^3 = \frac{8\pi^2 G}{15c^4}[(\varepsilon_{vac} + 3p_{vac})^2 R^5]. \tag{3.2}$$

We now solve this quadratic equation with respect to the pressure p_{vac} and get the following two solutions:

$$p_{vac,1} = -\frac{1}{3}\varepsilon_{vac} \tag{3.3}$$

and

$$p_{vac,2} = \frac{1}{3}(\frac{5c^4}{2\pi G R^2} - \varepsilon_{vac}). \tag{3.4}$$

Insertion of Eq. (3.3) or Eq. (3.4) into Eq. (2.3) results in both cases in one and the same differential equation for the energy density ε_{vac} given by:

$$\frac{d\varepsilon_{vac}}{dR}R + 2\varepsilon_{vac} = 0 \tag{3.5}$$

which has the unique solution:

$$\varepsilon_{vac} = \varepsilon_{vac,0}\frac{R_0^2}{R^2} \sim p_{vac,1,2} \tag{3.6}$$

with $\varepsilon_{vac,0}$ the vacuum energy density at a scale parameter R_0, e.g. at the present cosmic time t_0. Using $\varepsilon_{vac} = \rho_{vac}c^2$ we finally get from Eq. (3.6) for the associated cosmic mass density ρ_{vac} of a pure vacuum-energy-dominated universe which scales according to R^{-2}:

$$\rho_{vac} = \rho_{vac,0}\frac{R_0^2}{R^2}. \tag{3.7}$$

Similar results, however derived independently from very different theoretical reasons, have already been published by [Basilakos (2009)], [Solà (2013)], [Basilakos (2013)] and [Solà (2014)]. In these papers it has been discussed that strictly keeping to covariance requirements of the underlying general relativistic field equations one can allow for a time-dependence of the inherent cosmic vacuum energy density ρ_{vac} and, as a leading term, one should preferably consider the following time-dependence of the vacuum energy density $\rho_{\mathrm{vac}} = \rho_{\mathrm{vac},0} + \alpha \cdot H^2(t)$, where $H = H(t) = \dot{R}/R$ denotes the time-dependent Hubble constant within a Friedman-Lemaitre cosmology. As the above authors emphasize, this new setting will help solving many outstanding problems in the present-day cosmology like triggering a smooth transition from an initial inflationary expansion powered by very strong vacuum energy density into a present-day smooth inflation at very low vacuum energy densities of the order of $\rho_{\mathrm{vac},0} \simeq 10^{-29} g/cm^3$.

A similar attempt to subject the field equations to more general scale-invariance requirements has led [Scholz (2009)] on the basis of a Weylian scalar-tensor theory also to a term which acts equivalent to vacuum energy density and which is varying with $(1/R^2)$ exactly like derived in our above approach. The question may, however, come up here with concern to the justification of a scale-invariance requirement applied to the GRT field equations. Nevertheless, there are hints from many sides that a scale- or time-dependent vacuum energy term $\rho_{\mathrm{vac}} = \rho_{\mathrm{vac}}(t)$ seems to make much sense in cosmology.

4 Friedmann-Lemaître equations for a R^{-2}-scaling of ρ_{vac}

The Friedmann equations provide a relationship between the cosmic scale R, its first and second time derivatives $\dot{R}$ and $\ddot{R}$ on one hand, and the cosmic mass density ρ and its associated pressure p

on the other hand. In the following we investigate a pure vacuum energy filled universe with curvature $K = 0$. The Friedmann equations are then given by:

$$H^2(t) = \frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3}\rho_{vac} \tag{4.1}$$

and

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}(\rho_{vac}c^2 + 3p_{vac}) \tag{4.2}$$

with $H(t)$ the time dependent Hubble parameter. Insertion of the R^{-2}-dependent equivalent mass density of the vacuum energy given by Eq. (3.7) into Eq. (4.1) leads to:

$$H^2(t) = \frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3}\rho_{vac,0}\frac{R_0^2}{R^2} \tag{4.3}$$

which provides the following result for the expansion velocity $\dot{R}$ of the scaling factor R:

$$\dot{R} = \sqrt{\frac{8\pi G\rho_{vac,0}}{3}}R_0 = const. \tag{4.4}$$

and thus, if we require $R(t = 0) = 0$:

$$R = \sqrt{\frac{8\pi G\rho_{vac,0}}{3}}R_0 t. \tag{4.5}$$

We now look at the 2. Friedmann equation Eq. (4.2). The calculated pressure in eq. Eq. (3.3) results in a cosmic acceleration which is simply zero:

$$\begin{aligned}\ddot{R} = -\frac{4\pi G}{3c^2}(\rho_{vac}c^2 + 3p_{vac,1})R = \\ -\frac{4\pi G}{3c^2}(\rho_{vac}c^2 - 3\frac{1}{3}\rho_{vac}c^2)R = 0.\end{aligned} \tag{4.6}$$

However, the pressure in Eq. (3.4) leads to the following expression:

$$\begin{aligned}\ddot{R} = -\frac{4\pi G}{3c^2}(\rho_{vac}c^2 + 3\frac{1}{3}\frac{5c^4}{2\pi G R^2} - \\ 3\frac{1}{3}\rho_{vac}c^2)R = -\frac{10c^2}{3R}.\end{aligned} \tag{4.7}$$

The result of Eq. (4.7) is in discrepancy with the constant expansion velocity $\dot{R}$ in Eq. (4.4) which follows from the 1. Friedmann equation which itself does not depend on the pressure. Thus, since a constant $\dot{R}$ cannot be realized with Eq. (4.7), we can conclude that the pressure in Eq. (3.4) and its associated acceleration in Eq. (4.7) are of course mathematical solutions of our thermodynamical equations but not physical ones which are realized in a cosmos with a vacuum energy density which scales according to R^{-2} and which always leads to $\dot{R} = const.$, i.e. $\ddot{R} = 0$. With other words, the correlation between a vacuum energy density $\epsilon_{vac} \sim R^{-2}$ and its associated pressure p_{vac} is given by (equation of state):

$$p_{vac} = -\frac{1}{3}\epsilon_{vac}. \tag{4.8}$$

5 Consequences of the R^{-2}-scaling of ρ_{vac} and conclusions

With the results of the previous chapter for a matter-free, empty universe dominated by pure vacuum energy and with a curvature parameter $K = 0$ (i.e. a flat vacuum universe) we now look at the Hubble parameter $H(t)$ which is given for this universe by (see Eqs. (4.4) and (4.5):

$$H(t) = \frac{\dot{R}}{R} = \frac{\sqrt{\frac{8\pi G\rho_{vac,0}}{3}}R_0}{\sqrt{\frac{8\pi G\rho_{vac,0}}{3}}R_0 t} = \frac{1}{t} \tag{5.1}$$

and for the present cosmic time t_0 leads to $t_0 = 1/H_0(t_0) \approx 1,37 \cdot 10^{10} yrs$ with the presently accepted Hubble parameter $H_0 \approx 72km/s/Mpc$ (see Bennett et al. 2003).

Furthermore, we can now try to calculate the equivalent of the total, global vacuum energy content of the universe, i.e. the mass content M_{vac} of such an universe assuming that the extension of the visible universe is given by the so-called Hubble radius R_H, defined as that cosmic distance where the cosmic recession velocity $\dot{R}$ equals the velocity of light c and given by:

$$R_H = \frac{c}{H(t)} = ct \tag{5.2}$$

with $H(t)$ given by Eq. (5.1). Now, in addition Eq. (4.1) leads us to the cosmic density:

$$\rho_{vac} = \frac{3H^2}{8\pi G} = \frac{3}{8\pi G t^2} \tag{5.3}$$

which is nowadays ($t = t_0$):

$$\rho_{vac,0} = \frac{3H_0^2}{8\pi G} = \frac{3}{8\pi G t_0^2} \approx 10^{-26}\frac{kg}{m^3}. \tag{5.4}$$

Hence we can express the present vacuum mass of the universe by:

$$M_{vac} = \frac{4\pi}{3}\rho_{vac,0}R_H^3 = \frac{4\pi}{3}\frac{3}{8\pi G t_0^2}c^3 t_0^3 = \frac{c^3}{2G}t_0 \approx 10^{53}kg \approx 10^{80}m_p \tag{5.5}$$

with m_p as the mass of the proton. Interestingly, the Eqs. (5.4) and (5.5) show well-known numbers, quite familiar to nowadays astronomers, namely just numbers for the presently assumed critical mass density of our universe and the present mass content of the visible universe, respectively. This may in first glance appear to be completely casual and be highly astonishing, since with the above we calculated density and mass of a cosmic vacuum on the basis of a R^{-2}-scaling vacuum energy density, while the numbers that we got are typical for the matter content of our present universe.

These above results are, however, not judged by the authors of this paper to be an numerical artifact, but may have the following important reason: We can take Eq. (3.7) to calculate the equivalent mass density of the vacuum energy density of the very early universe, i.e. at the Planck time t_p or the Planck length $R_H(t_p) = r_p = ct_p$, thereby expressing the reference scale R_0 by the present Hubble radius $R_{H,0} = ct_0$ (according to Eq. (5.2)) and get:

$$\rho_{vac}(r_p) = \rho_{vac}(t_p) = \rho_{vac,0}\frac{R_{H,0}^2}{r_p^2} = \rho_{vac,0}\frac{ct_0^2}{ct_p^2} = \rho_{vac,0}\frac{t_0^2}{t_p^2}. \tag{5.6}$$

If we now substitute $\rho_{vac,0}$ by $3/8\pi G t_0^2$ (ref. Eq. (5.4)) then Eq. (5.6) can be written as:

$$\rho_{vac}(r_p) = \rho_{vac}(t_p) = \frac{3}{8\pi G t_0^2}\frac{t_0^2}{t_p^2} = \frac{3}{8\pi G t_p^2}. \tag{5.7}$$

When we replace the Planck time $t_p = r_p/c = \sqrt{\hbar G/c^5}$ we finally get the following formula:

$$\rho_{vac}(r_p) = \rho_{vac}(t_p) = \frac{3}{8\pi}\frac{c^5}{\hbar G^2} = \rho_p \tag{5.8}$$

which is identical to the Planck density ρ_p defined by the ratio of a half Planck mass $\frac{1}{2}m_p = \frac{1}{2}\sqrt{\hbar c/G}$ and the Planck volume $\frac{4\pi}{3}r_p^3$ with the Planck length $r_p = \sqrt{\hbar G/c^3}$. This means that the equivalent vacuum mass density which scales according to R^{-2} in our model can be described as a scaling Planck density ρ_p. In fact, we can re-write Eq. (5.3) by replacing the factor $3/8\pi G$ using Eq. (5.8) and get:

$$\rho_{vac}(t) = \frac{3}{8\pi G t^2} = \rho_p \frac{\hbar G/c^5}{t^2} = \rho_p \frac{t_p^2}{t^2} \tag{5.9}$$

where the Planck time $t_p = \sqrt{\hbar G/c^5}$ is now the reference time. The ratio $\rho_{vac,0}/\rho_p$ is then simply given by:

$$\frac{\rho_{vac,0}}{\rho_p} = \frac{t_0^2}{t_p^2} \approx \cdot 10^{-122} \tag{5.10}$$

and also is a well-known discrepancy factor with respect to the ratio of the present vacuum mass density on one hand and the theoretical value of the vacuum mass density that follows from field-theoretical calculations on the other hand ([Zeldovich (1968)] and [Weinberg (1989)]). Thus we can conclude, that this discrepancy vanishes for a vacuum energy density that scales according to R^{-2} as shown in this paper.

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