Bianchi Type-IX Cosmological Model With Perfect Fluid in *f(R)* Theory of Gravity

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ABSTRACT

Bianchi type-IX space-time have been obtained when universe is filled with perfect fluid in $f(R)$ theory of gravity. Here, it is considered in the framework of $f(R)$ theory of gravity when the source for energy momentum tensor is a perfect fluid. The cosmological model is obtained by using the condition that expansion scalar (θ) proportional to the shear scalar (σ).The physical and geometrical properties of the model ~~are also discussed~~. It is observed that the scale factors and volume of the model vanishes at initial epoch and increases with the passage of time representing an expanding universe. We hope to expand our model y to explain the structure formation and accelerated expansion for the early universe.

Keywords: $f(R)$ gravity, Bianchi Type-IX space-time

1. INTRODUCTION

Cosmological observations in the late 90's from different sources such as Cosmic Microwave Background Radiations (CMBR) and Supernovae (SN Ia) surveys indicate that the universe consist of 4% ordinary matter, 20% dark matter (DM) and 76% dark energy (DE) [1-4]. The DE has large negative pressure while the pressure of DM is negligible. Wald [5] has distinguished DM and DE and clarified that ordinary matter and DM satisfy the strong energy conditions, whereas DE does not. The DE resembles with a cosmological constant and scalar fields. The scalar field is provided by the dynamically changing DE including quintessence, k-essence, tachyon, phantom, ghost condensate and quintom etc. The study of high redshift supernova experiments [6-8], CMBR [9-10], Large scale structure [11] and recent evidences from observational data [12-14] suggest that the universe is not only expanding but also accelerating.

There are two major approaches according to the problem of accelerating expansion. One is to introduce DE component in the universe and study its effects. Other alternative is to modify general relativity termed as a modified gravity approach. We are interested in a second alternative~~one~~. After the introduction of General Relativity (GR) in 1915, questions related to its limitations were in discussion. Einstein pointed out that Mach's principle is not substantiated by general relativity. Several attempts have been made to generalize the general theory of gravitation by incorporating Mach's principle and other desired features, which were lacking in the original theory. Alternative theories of gravitation have been proposed to Einstein's theory to incorporate certain desirable features in the general theory. In the last decades, as an alternative to general relativity, scalar tensor theories and modified theories of gravitation have been proposed. The most popular amongst them are Brans-Dicke [15], Nordtvedt [16], Sen [17], Sen and Dunn [18], Wagonar [19], Saez-Ballester [20] etc. Recently, $f(R)$ gravity and $f(R,T)$ gravity theories have much importance amongst the modified theories of gravity because these theories are supposed to provide a natural gravitational alternative to dark energy. Amongst the various modifications, $f(R)$ theory of gravity is treated most suitable due to cosmologically important $f(R)$ models. In $f(R)$ gravity, the Lagrangian density f is an arbitrary function of R [15, 21-23]. The model with $f(R)$ gravity can laid to the accelerated expansion of the universe. A generalization of $f(R)$ modified theory of gravity was proposed by Takahashi and Soda [24] by including explicit coupling of an arbitrary function of the Ricci Scalar R with the matter Lagrangian density L_m. There are two formalism in deriving field equations from the action in $f(R)$ gravity. The first is the standard metric formalism in which the field equations are derived by the variation of the action with respect to the metric tensor $g_{\mu\nu}$. The second is the Palatini formalism. Maeda [25] have investigated Palatini formulation of the non-minimal geometry-coupling models. Multamaki and Vilja [26] obtained spherically symmetric solutions of modified field equations in $f(R)$ theory of gravity. Akbar and Cai [27] studied $f(R)$ theory of gravity action is a nonlinear function of the curvature scalar R. Nojiri and Odinstove [28-30] derived that a unification of the early time inflation and late time acceleration is allowed in $f(R)$ theory. Ananda, Carloni and Dunsby [31] studied structure growth in $f(R)$ theory with dust equation of state. Sharif and Shamir [32] and Sharif [33] have studied the vacuum solutions of Bianchi type-I, V and VI space-times. Sharif and

Shamir [34] and Sharif and Kausar [35] obtained the non-vacuum solutions of Bianchi type-I, III and V space-times in $f(R)$ theory of gravity. Adhav [36, 37] have investigated Kantowski-Sachs string cosmological model and Bianchi type-III cosmological model with perfect fluid in $f(R)$ gravity. Singh and Singh [38] have obtained functional form of $f(R)$ with power-law expansion in Bianchi type-I space-times. Recently Jawad and Chattopadhyay [39] have investigated new holographic dark energy in $f(R)$ Horava Lifshitz gravity. Rahman et al. [40] have obtained non-commutative wormholes in $f(R)$ gravity with Lorentzian distribution.

Motivated by the above investigations, in this paper an attempt is made to study Bianchi type-IX space-time when universe is filled with perfect fluid in $f(R)$ theory of gravity with standard metric formalism. Bianchi type-IX space-time are of vital importance in describing cosmological models at the early stages of evolution of the universe. This work is organized as follows: In Section 2, $f(R)$ gravity formalism is presented. In Section 3, the model and field equations have been presented. The field equations have been solved in Section 4. The physical and geometrical behaviors of the two models have been discussed in Section 5. In Section 6, concluding remarks have been expressed.

2. $f(R)$ GRAVITY FORMALISM:

The action $f(R)$ gravity is given by

$$S = \int \sqrt{-g}\left(\frac{1}{16\pi G} f(R) + L_m\right) d^4x \,. \tag{1}$$

Here $f(R)$ is a general function of the Ricci scalar R and L_m is the matter Lagrangian. The corresponding field equations of the $f(R)$ gravity are found by varying the action with respect to the metric $g_{\mu\nu}$:

$$F(R)R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} - \nabla_\mu \nabla_\nu F(R) + g_{\mu\nu} \Box F(R) = kT_{\mu\nu} \,, \tag{2}$$

where $F(R) = \frac{d}{dR} f(R)$, $\Box \equiv \nabla^\mu \nabla_\nu$, ∇_μ is the covariant derivative and $T_{\mu\nu}$ is the standard matter energy-momentum tensor derived from the Lagrangian L_m.

Taking trace of the above equation (with $k=1$), we obtain

$$F(R)R - 2f(R) + 3\Box F(R) = T \,. \tag{3}$$

On simplification, equation (3) leads to

$$f(R)=\frac{F(R)R+3\nabla^{\mu}\nabla_{\mu}F-T}{2}. \quad (4)$$

3. METRIC AND FIELD EQUATIONS:

Bianchi type-IX metric is considered in the form,

$$ds^2=-dt^2+a^2dx^2+b^2dy^2+\left(b^2\sin^2 y+a^2\cos^2 y\right)dz^2-2a^2\cos y\,dxdz, \quad (5)$$

where a, b are scale factors and are functions of cosmic time t.

The Ricci scalar for Bianchi type-IX model is given by

$$R=-2\left[\frac{\ddot{a}}{a}+2\frac{\ddot{b}}{b}+2\frac{\dot{a}}{a}\frac{\dot{b}}{b}+\frac{\dot{b}^2}{b^2}+\frac{1}{b^2}-\frac{a^2}{4b^4}\right]. \quad (6)$$

The energy momentum tensor for the perfect fluid is given by

$$T_{ij}=(\rho+p)u_iu_j-pg_{ij}, \quad (7)$$

satisfying the barotropic equation of state

$$p=\gamma\rho,\ 0\le\gamma\le 1, \quad (8)$$

where ρ is the energy density and p is the pressure of the fluid.

In co-moving coordinate system

$$T_1^1=T_2^2=T_3^3=-p,\ .T_4^4=\rho,\quad T=\rho-3p. \quad (9)$$

With the help of equations (7) to (9), the field equations (2) for the metric (5) are

$$\left(\frac{\ddot{a}}{a}+2\frac{\ddot{b}}{b}\right)F+\frac{1}{2}f(R)+\left(\frac{\dot{a}}{a}+2\frac{\dot{b}}{b}\right)\dot{F}=-\rho, \quad (10)$$

$$\left(\frac{\ddot{a}}{a}+2\frac{\dot{a}}{a}\frac{\dot{b}}{b}+\frac{a^2}{2b^4}\right)F+\frac{1}{2}f(R)+\ddot{F}+2\frac{\dot{b}}{b}\dot{F}=p, \quad (11)$$

$$\left(\frac{\ddot{b}}{b}+\frac{\dot{b}^2}{b^2}+\frac{\dot{a}}{a}\frac{\dot{b}}{b}+\frac{1}{b^2}-\frac{a^2}{2b^4}\right)F+\frac{1}{2}f(R)+\ddot{F}+\left(\frac{\dot{a}}{a}+\frac{\dot{b}}{b}\right)\dot{F}=p, \quad (12)$$

where the overdot $(\dot{})$ denotes the differentiation with respect to *t*.

4. SOLUTIONS OF FIELD EQUATIONS:

The field equations (10) to (12) are highly non-linear differential equations in five unknowns a, b, p, ρ, F. Hence to obtain a determinate solution of the system we take the expansion scalar (θ) is proportional to the shear scalar (σ) (Collin et al. [41]), which leads to

$$a = b^m \,, (m \neq 1)\,, \tag{13}$$

where m is proportionality constant.

Also the power law relation between scale factor (A) and scalar field (F) [37, 42-43] has been given by

$$F \,\alpha\, A^n \,, \tag{14}$$

where n is arbitrary constant and A is average scale factor.

Equation (14) leads to

$$F = K\, A^n \,, \tag{15}$$

where K is proportionality constant.

With the help of equation (13), equation (15) reduces to

$$F = K\, b^{\frac{(m+2)n}{3}} \,. \tag{16}$$

Subtraction of equation (10) from (11), (10) from (12) respectively and dividing the result by F gives

$$2\frac{\dot{a}}{a}\frac{\dot{b}}{b} - 2\frac{\ddot{b}}{b} + \frac{a^2}{2b^4} + \frac{\ddot{F}}{F} - \frac{\dot{a}}{a}\frac{\dot{F}}{F} = \frac{p+\rho}{F}\,, \tag{17}$$

$$\frac{\dot{a}}{a}\frac{\dot{b}}{b} - \frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} - \frac{a^2}{2b^4} + \frac{\ddot{F}}{F} - \frac{\dot{b}}{b}\frac{\dot{F}}{F} = \frac{p+\rho}{F}\,. \tag{18}$$

Subtraction of equation (18) from equation (17) yields

$$\frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} - \frac{\dot{a}}{a}\frac{\dot{F}}{F} + \frac{\dot{b}}{b}\frac{\dot{F}}{F} - \frac{1}{b^2} - \frac{\dot{b}^2}{b^2} + \frac{a^2}{b^4} = 0\,. \tag{19}$$

With the help of equation (13) and (16), equation (19) leads to

$$\frac{\ddot{b}}{b} + \frac{(3m^2 - m^2n - mn + 2n - 3)}{3(m-1)}\frac{\dot{b}^2}{b^2} - \frac{1}{(m-1)b^2} + \frac{b^{2m-4}}{(m-1)} = 0\,. \tag{20}$$

On simplification, equation (20) reduces to

$$\frac{d}{db}(\dot{b}^2) + \frac{2(3m^2 - m^2n - mn + 2n - 3)}{3(m-1)b}(\dot{b}^2) = \frac{1}{(m-1)}\left(2b^{-2} - 2b^{2m-3}\right). \tag{21}$$

Integrating equation (21)

$$\dot{b} = \sqrt{\frac{3}{(3m^2 - m^2n - mn + 2n - 3)} - \frac{3}{(6m^2 - m^2n - mn - 6m + 2n)}b^{2(m-1)}}\,. \tag{22}$$

Using equations (13) and (22), equation (5) reduces to

$$ds^2 = \left\{ \begin{array}{l} -\dfrac{1}{\left\{ \dfrac{3}{(3m^2 - m^2 n - mn + 2n - 3)} - \dfrac{3}{(6m^2 - m^2 n - mn - 6m + 2n)} b^{2(m-1)} \right\}} db^2 \\ + b^{2m} dx^2 + b^2 dy^2 + \left(b^2 \sin^2 y + b^{2m} \cos^2 y \right) dz^2 - 2b^{2m} \cos y dx dz \end{array} \right\} . \quad (23)$$

Using transformations $b = T, x = X, y = Y, z = Z$, equation (23) leads to

$$ds^2 = \left\{ \begin{array}{l} -\dfrac{1}{\left\{ \dfrac{3}{(3m^2 - m^2 n - mn + 2n - 3)} - \dfrac{3}{(6m^2 - m^2 n - mn - 6m + 2n)} b^{2(m-1)} \right\}} dT^2 \\ + T^{2m} dX^2 + T^2 dY^2 + \left(T^2 \sin^2 Y + T^{2m} \cos^2 Y \right) dZ^2 - 2b^{2m} \cos Y dX dZ \end{array} \right\} . \quad (24)$$

5. SOME PHYSICAL PROPERTIES OF THE MODEL:

For the cosmological model (24), the physical quantities spatial volume V, Hubble parameter H, expansion scalar θ, mean anisotropy parameter A_m, shear scalar σ^2, energy density ρ are obtained as follows:

Spatial volume,

$$V = T^{m+2} . \quad (25)$$

Hubble parameter,

$$H = \frac{(m+2)}{3T} \left(\frac{3}{(3m^2 - m^2 n - mn + 2n - 3)} - \frac{3}{(6m^2 - m^2 n - mn - 6m + 2n)} T^{2(m-1)} \right)^{1/2} . \quad (26)$$

Expansion scalar,

$$\theta = \frac{(m+2)}{T} \left(\frac{3}{(3m^2 - m^2 n - mn + 2n - 3)} - \frac{3}{(6m^2 - m^2 n - mn - 6m + 2n)} T^{2(m-1)} \right)^{1/2} . \quad (27)$$

Mean Anisotropy Parameter,

$$A_m = \frac{2(m-1)^2}{(m+2)^2} = \text{constant } (\neq 0 \text{ for } m \neq 1) . \quad (28)$$

Shear scalar,

$$\sigma^2 = \left\{ \frac{(m-1)^2}{(3m^2 - m^2 n - mn + 2n - 3)} T^{-2} - \frac{(m-1)^2}{(6m^2 - m^2 n - mn - 6m + 2n)} T^{2(m-2)} \right\}. \tag{29}$$

$$\frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(m+2)^2} = \text{constant } (\neq 0) \text{, for } m \neq 1. \tag{30}$$

Using equations (8), (16) and (22) in equation (10), the energy density is obtained as

$$\rho = \frac{K}{2(1+\gamma)} T^{\frac{mn+2n-6}{3}} \left(\begin{array}{l} 1 + \dfrac{9(1+4m-m^2) + (mn+2n)(2mn-3m+4n-9)}{3(3m^2 - m^2 n - mn + 2n - 3)} \\ - \dfrac{\left[\begin{array}{l} 9(1+4m-m^2) \\ +(mn+2n-3m+3)(2mn-3m+4n-9) \end{array} \right]}{3(6m^2 - m^2 n - mn - 6m + 2n)} T^{2(m-1)} \end{array} \right). \tag{31}$$

From equation (6) we obtain

$$R = \left(\begin{array}{l} \left[1 + \dfrac{3(1+m+m^2)}{(3m^2 - m^2 n - mn + 2n - 3)} \right] - \left[\dfrac{3(1+m+m^2)}{(6m^2 - m^2 n - mn - 6m + 2n)} + \dfrac{2(m+3)}{2(m-1)} \right. \\ \left. \dfrac{2(m+2)(3m^2 - m^2 n - mn + 2n - 3)}{(m-1)(6m^2 - m^2 n - mn - 6m + 2n)} \right] T^{2(m-2)} \end{array} \right). \tag{32}$$

From equation (4) the function of Ricci scalar $f(R)$ leads to

$$f(R) = \left\{ \begin{array}{l} \left[\begin{array}{l} 1 + \dfrac{3(1+4m-m^2) + (mn+2n)(mn+2n-3)}{(3m^2 - m^2 n - mn + 2n - 3)} - \dfrac{(1-3\gamma)}{2(1+\gamma)} \\ \left(1 + \dfrac{9(1+4m-m^2) + (mn+2n)(2mn-3m+4n-9)}{3(3m^2 - m^2 n - mn + 2n - 3)} \right) \end{array} \right] \dfrac{K}{2} T^{\frac{(m+2)n-6}{3}} - \\ \left[\begin{array}{l} \dfrac{3(1+m+m^2) + (mn+2n)(mn+2n-3)}{(6m^2 - m^2 n - mn - 6m + 2n)} + \dfrac{(3m+9-2mn-4n)}{2(m-1)} \\ + \dfrac{(mn+2m+2n+4)(3m^2 - m^2 n - mn + 2n - 3)}{(m-1)(6m^2 - m^2 n - mn - 6m + 2n)} \\ - \dfrac{(1-3\gamma)}{2(1+\gamma)} \left(\dfrac{9(1+4m-m^2) + (mn+2n-3m+3)(2mn-3m+4n-9)}{3(6m^2 - m^2 n - mn - 6m + 2n} \right) \end{array} \right] \dfrac{K}{2} T^{\frac{(m+2)n+6(m-2)}{3}} \end{array} \right\}$$

, (33)

which clearly indicates that $f(R)$ is written in terms of T, which is true as $f(R)$ depends upon T.

By inserting the value of R from equation (32) in equation (33), $f(R)$ reduces to a function of R.

For a special case when $m = n = 2$, $f(R)$ turns out to be

$$f(R)=\frac{K}{3(1+\gamma)}\left(\frac{88}{59+4R}\right)\left[-41+15\gamma+\frac{220(2-3\gamma)}{59+4R}\right]. \qquad (34)$$

This gives $f(R)$ only as a function of R .

6. CONCLUSION:

Bianchi type-IX cosmological model have been obtained when universe is filled with perfect fluid in $f(R)$ theory of gravity. The model obtained has singularity at $T=0$ and the physical parameters H , θ , σ^2 are infinite at $T=0$ as well. It is observed that the scale factors and volume of the model vanishes at initial epoch and increases with the passage of time representing expanding universe. From equation (26) and (28) the mean anisotropy parameter A_m is constant and $\frac{\sigma^2}{\theta^2}(\neq 0)$ is also constant, hence the model is anisotropic throughout the evolution of the universe except at $m=1$ *i.e.* the model does not approach isotropy.

It is worth to mention that, the model obtained is point type singular, expanding, shearing, that is non-rotating and does not approach isotropy for large T . We hope that our model will be useful in the study of structure formation in the early universe and an accelerating expansion of the universe at present.

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