Original Research Article

Charge Radii of B and D mesons in a Quark Model with two loop static potential

Abstract

We study the effects of two loop static potential on the charge radii of heavy light flavoured mesons in the Improved QCD Inspired Quark model. The effect of a constant term "c" in the Cornell potential is also studied in the computation of the charge radii of heavy-light mesons.

Keywords: Two loop static potential and charge radii.
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1 Introduction

The study of hadron wavefunctions and their phenomenology is one of the important topics in QCD [1]. Since the classical work of De Rujula et al [2], such studies in non-relativistic potential approach have been made in several models [3, 4, 5, 6, 7, 8, 9, 10] with considerable success.Inspired by the success of this approach, the present authors have been pursuing a specific QCD Inspired Quark model [11, 12, 13, 14, 15] which has undergone successive and phenomenological developments in various stages : in earlier version [11, 12, 13, 14] significant confinement effects could not be introduced due to perturbative constraints, while in the later version [15] it could be incorporated.

The present paper reports the results of charge radii,masses and decay constants of heavy light mesons within the improved version of the model. The corresponding results for the Isgur-Wise function and its slope and curvature have already been reported in the recent communication [15].

In section 2 , we discuss the formalism , while in section 3 we summarise the results. Section 4 contains summary and conclusion .

2 Formalism

2.1 The Improved QCD Inspired Quark Model.

The essential features of the improved model has already been reported in ref [15]. The wave function in this improved model is given by

$$\psi_{rel+conf}(r) = \frac{N'}{\sqrt{\pi a_0^3}} \left(C'(c) - \frac{1}{2}\mu b a_0 r^2 \right) \left(\frac{r}{a_0} \right)^{-\epsilon} e^{-\frac{r}{a_0}} \tag{1}$$

where

$$N' = \frac{2^{\frac{1}{2}}}{\left[2^{2\epsilon} \left(\Gamma(3-2\epsilon) \left\{ C'(c) \right\}^2 - \frac{1}{4}\mu b a_0^3 \Gamma(5-2\epsilon) C'(c) + \frac{1}{64}\mu^2 b^2 a_0^6 \Gamma(7-2\epsilon) \right) \right]^{\frac{1}{2}}} \tag{2}$$

is the normalisation constant; ϵ and a_0 are defined by

$$\epsilon = 1 - \sqrt{1 - \frac{4}{3}\alpha_S} \tag{3}$$

and

$$a_0 = \left(\frac{4}{3\mu\alpha_{\overline{MS}}} \right)^{-1} \tag{4}$$

respectively, with

$$C'(c) = 1 + cA_0\sqrt{\pi a_0^3} \tag{5}$$

A_0 being the undetermined factor appearing in the series solution of the Schrödinger equation and μ being the reduced mass.

From equation (5), if A_0 and/or c is set equal to zero, the effect of c disappears in the formalism as was the case for [11, 12, 13, 14, 16]. From equations (1) and (2), it is also evident that in the limit $b \to 0$, the effect of $C'(c)$ and hence of c completely disappears in the wave function.

2.2 Two loop static potential in V scheme

As has already been described in [14, 15], the V-scheme (V denoting potential) [17, 18, 19, 20] defines the QCD coupling constant in terms of a potential, taking into account the higher order effects of QCD which are then expressed as a power series in the coupling constant $\alpha_{\overline{MS}}$ in $\overline{MS}$ scheme. In the V scheme α_V represents the effective coupling constant which incorporates the entire radiative corrections into its definition.

2.3 Form Factor and Charge Radii.

The elastic charge form factor for a charged system of point quarks has the form [21, 22]

$$eF\left(Q^2\right) = \sum_{i=1}^{2} \frac{e_i}{Q_i} \int_0^\infty r \mid \Psi(r) \mid^2 \sin(Q_i r)\, dr \tag{6}$$

Using equation (1) in (6) and integrating over r

$$eF\left(Q^2\right) = \frac{N'^2}{a_0 2^{-2\epsilon}} \sum_{i=1}^{2} \frac{e_i}{Q_i} \Bigg[C'^2 \Gamma\left(2-2\epsilon\right) \sin\left[\left(2-2\epsilon\right)\theta_i\right] \left(1+\left(\frac{1}{4}a_0^2 Q_i^2\right)\right)^{(\epsilon-1)} - \frac{1}{4}\mu b a_0^3 C' \Gamma\left(4-2\epsilon\right) \sin\left[\left(4-2\epsilon\right)\theta_i\right] \left(1+\left(\frac{1}{4}a_0^2 Q_i^2\right)\right)^{(\epsilon-2)} + \frac{1}{64}\mu^2 b^2 a_0^6 \Gamma\left(6-2\epsilon\right) \sin\left[\left(6-2\epsilon\right)\theta_i\right] \left(1+\left(\frac{1}{4}a_0^2 Q_i^2\right)\right)^{(\epsilon-3)} \Bigg] \tag{7}$$

where

$$\theta_i = \sin^{-1} \frac{Q_i}{\left(\frac{4}{a_0^2}+Q_i^2\right)^{\frac{1}{2}}}. \tag{8}$$

In evaluating (7) we use the approximation in (8) to be

$$\sin^{-1} x \approx x + \frac{x^3}{6} + \frac{3x^5}{40} \tag{9}$$

with

$$x = \frac{Q_i}{\left(\frac{4}{a_0^2}+Q_i^2\right)^{\frac{1}{2}}}. \tag{10}$$

We also use

$$\sin y = y - \frac{y^3}{3!} + \frac{y^5}{5!} \tag{11}$$

with

$$y = (2-2\epsilon)\theta_i. \tag{12}$$

As a result of these improved approximations, the expression for $eF(Q^2)$ in equation (7) becomes

$$eF\left(Q^2\right) = \frac{N_1^2}{a_0 2^{-2\epsilon}} \sum_{i=1}^{2} e_i \Bigg[\left(1+\frac{1}{4}a_0^2 Q_i^2\right)^{(\epsilon-1)} \left(\left(2-2\epsilon\right) X_i - \frac{1}{6}\left(2-2\epsilon\right)^3 Q_i^2 X_i^3 + \frac{1}{120}\left(2-2\epsilon\right)^5 Q_i^4 X_i^5\right) C'^2 \Gamma\left(2-2\epsilon\right) - \frac{1}{4}\mu b a_0^3 \left(1+\frac{1}{4}a_0^2 Q_i^2\right)^{(\epsilon-2)} \left(\left(4-2\epsilon\right) X_i - \frac{1}{6}\left(4-2\epsilon\right)^3 Q_i^2 X_i^3 + \frac{1}{120}\left(4-2\epsilon\right)^5 Q_i^4 X_i^5\right) C' \Gamma\left(4-2\epsilon\right) + \frac{1}{64}\mu^2 b^2 a_0^6 \left(1+\frac{1}{4}a_0^2 Q_i^2\right)^{(\epsilon-3)} \left(\left(6-2\epsilon\right) X_i - \frac{1}{6}\left(6-2\epsilon\right)^3 Q_i^2 X_i^3 + \frac{1}{120}\left(6-2\epsilon\right)^5 Q_i^4 X_i^5\right) \Gamma\left(6-2\epsilon\right) \Bigg] \tag{13}$$

where

$$X_i = \frac{3Q_i^4}{40}\left(\frac{4}{a_0^2}+Q_i^2\right)^{-\frac{5}{2}} + \frac{Q_i^2}{6}\left(\frac{4}{a_0^2}+Q_i^2\right)^{-\frac{3}{2}} + \left(\frac{4}{a_0^2}+Q_i^2\right)^{-\frac{1}{2}}, \tag{14}$$

N_1 being the normalization constant,

$$N_1 = 2^{\frac{1}{2}} \left(2^{2\epsilon}\left(\left(2-2\epsilon\right) C'^2 \Gamma\left(2-2\epsilon\right) - \frac{1}{4}\mu b a_0^3 \left(4-2\epsilon\right) C' \Gamma\left(4-2\epsilon\right) + \frac{1}{64}\mu^2 b^2 a_0^6 \left(6-2\epsilon\right) \Gamma\left(6-2\epsilon\right)\right)\right)^{-\frac{1}{2}}. \tag{15}$$

The average charge radii square for the mesons are obtained from the relation

$$\left\langle r^2\right\rangle = -6\frac{dF(Q^2)}{dQ^2}\mid_{Q^2=0} . \tag{16}$$

Using (13) in (16), one obtains for the mesons having quark masses m_i and m_j, and charges e_i and e_j respectively ,

$$\left\langle r^2\right\rangle = a_0^2\left[e_i\left(1+\frac{m_i}{m_j}\right)^{-2}+e_j\left(1+\frac{m_j}{m_i}\right)^{-2}\right]\left(C'^2\Gamma\left(2-2\epsilon\right)\left(\frac{(2-\epsilon)}{2}+\frac{(2-\epsilon)^3}{4}+\frac{3\left(2-\epsilon\right)\left(\epsilon-1\right)}{2}\right)-\frac{1}{4}\mu b a_0^3 C'\Gamma\left(4-2\epsilon\right)\left(\frac{(4-2\epsilon)}{2}+\frac{(4-2\epsilon)^3}{4}+\frac{3\left(4-2\epsilon\right)\left(\epsilon-2\right)}{8}\right)+\frac{1}{64}\mu^2b^2a_0^6\Gamma\left(6-2\epsilon\right)\left(\frac{(6-2\epsilon)}{2}+\frac{(6-2\epsilon)^3}{4}+\frac{3\left(6-2\epsilon\right)\left(\epsilon-3\right)}{128}\right)\right)\left(\left(2-2\epsilon\right)C'^2\Gamma\left(2-2\epsilon\right)-\frac{1}{4}\mu b a_0^3\left(4-2\epsilon\right)C'\Gamma\left(4-2\epsilon\right)+\frac{1}{64}\mu^2b^2a_0^6\left(6-2\epsilon\right)\Gamma\left(6-2\epsilon\right)\right)^{-1} . \tag{17}$$

3 Result and Discussion

3.1 Relationship between $\alpha_{\overline{MS}}$ and α_V

Taking $n_f = 4$ and $n_f = 5$ with the corresponding value of $\alpha_{\overline{MS}}$ at the scale of the c- and b- quark mass, $\alpha_{\overline{MS}}(m_c) = 0.39$ and $\alpha_{\overline{MS}}(m_b) = 0.22$ respectively [23], the value of $\alpha_V(\frac{1}{r^2})$ for three choices of $\widetilde{\mu}$ are calculated and shown in Table 1 of [14] . It shows that the two loop potential invariably scales up the effective coupling constant from $\alpha_{\overline{MS}}$ to α_V. The increase in percentage is also shown in Table 1. It is much more for c-quark than for b-quark. The values of the running coupling constant for B and D mesons are $\alpha_{\overline{MS}} = 0.22$ and $\alpha_{\overline{MS}} = 0.39$ respectively [23]. The two loop calculations raise these values for B and D mesons to $\alpha_V = 0.261$ and $\alpha_V = 0.625$ respectively as shown in Table 1, and are in good agreement with our results of [11] for D mesons. For B mesons however, the boost given by two loop results is insufficient.

3.2 Effect of the parameter c in the analysis

In the previous studies on this model [11, 12, 13, 14, 16], the undetermined factor appearing in the series solution of the Schrödinger equation A_0, (as occured in equation (5)), was set equal to zero and the effect of c disappeared altogether in the formalism. In the previous studies, a very small value of the confinement parameter b could only be accomodated due to the perturbative constaints. However, in the present analysis, the standard quarkonium spectroscopic result [24, 25] $b = 0.183GeV^2$ can also be accomodated by a suitable choice of c (taking $A_0 = 1$), since the new perturbative constraints given by equations (12) and (13) of [15] do not prohibit large values of $C'(c)$. As mentioned in [15], since the reduced masses of mesons are about 1 GeV or less, a very large value of the parameter c occuring in the Hamiltonian given in equation (12) of [11] would most probably not be natural.

3.3 Charge radii

The mean square charge radii of the heavy pseudoscalar mesons have not been measured yet . Using equation (17), we calculate the square of their charge radii $\langle r^2 \rangle_F$ for finite heavy quark masses in fm^2 and compare our results with Hwang [9] . We show our results for the mean square charge radii of the heavy pseudoscalar mesons in Table 1, taking $b = 0.183GeV^2$ and $c = 1GeV$ with $\alpha_V = 0.261$ for the B mesons, and in Table 2 with $\alpha_V = 0.65$ for the D mesons. Predictions for D mesons agree well with Hwang [9], but for B mesons, (with $\alpha_V = 0.261$), the corresponding predictions overshoot by a factor of 2 - 3 . This anomalous feature can be overcome only if α_V for B mesons is higher, say $\alpha_V \sim 0.6$.

3.4 Conclusions

We have calculated the charge radii of heavy light flavoured mesons with two loop static potential within a QCD inspired quark model pursued by us in recent years. We have also improved our earlier calculations [11, 13, 14] as substantial confinement effects can now be accommodated in contrast to the previous version of the model. Relativistic effects have been estimated in the wave function at the origin by a procedure analogous to the hydrogen atom in QED. Before conclusion, we also make a few comments.

The approach of the earlier version ($b \sim 0$) could be justified when the two constituent quarks are very heavy and their motion is restricted to the small area around the origin, such as in the case of charmonium or bottonium. For significant confinement effect ($b > 0$), no such restriction exists and is presumably applicable to the heavy-light mesons as well, as we have done in the present paper. The approach can be further tested for the excited heavy-light mesons and for their mass differences.

As we are considering only pseudo scalar mesons, the spin-spin interactions in equation (11) of [11] which give mass splitting between pseudoscalar and vector mesons is neglected. By redefining the c-term, this can be included.

In the present analysis, our results agree with current experimental data as well as with those of Hwang[9] for a value of the coupling constant at around α=0.6 to 0.65 instead of the running coupling constants of $\alpha_{\overline{MS}} = 0.39$ for D mesons and $\alpha_{\overline{MS}} = 0.22$ for B mesons. Such higher value of strong coupling constant can be generated in the V-scheme with $O\left(\alpha_S^3\right)$ terms for D mesons but falls short for B mesons. As noted earlier [14], this presumably indicates large flavour dependent higher order effects of $O\left(\alpha_S^3\right)$ for B mesons.

Although the present analysis is an improvement of the earlier work of [11, 13, 14] in the sense that significant confinement as well as higher order effects in α_S through V-scheme have been incorporated, it still falls short in explaining correctly the properties of mesons with b-quarks.

One plausible way is presumably to treat the Coulomb potential as perturbation to its confinement counterpart [26] in V-scheme. Such a possibility is currently under study.

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Table 1: Values of Mean Square Charge Radii of the Heavy-Light Pseudoscalar D Mesons in fm^2 for finite quark masses $\langle r^2 \rangle_F$ in the V scheme with $c = 1GeV$ for $b = 0.183GeV^2$ with $\alpha_V = 0.625$ and $A_0 = 1$ in comparison to the predictions of Hwang [9].

	ϵ_V	D_S^+	D^0	D^+
$\langle r^2 \rangle_F$, this work,	0.592	0.077	-0.302	0.172
$\langle r^2 \rangle_F$[9]	-	0.124	-0.304	0.180

Table 2: Values of Mean Square Charge Radii of the Heavy-Light Pseudoscalar B Mesons in fm^2 for finite quark masses $\langle r^2 \rangle_F$ in the V scheme with $c = 1GeV$ for $b = 0.183GeV^2$ and $A_0 = 1$ for both $\alpha_V = 0.261$ and $\alpha_V = 0.6$ in comparison to the predictions of Hwang [9].

	α_V	ϵ_V	B_C^+	B_S^0	B^0	B^+
$\langle r^2 \rangle_F$, this work	0.261	0.193	0.565	-5.401	-13.651	27.489
$\langle r^2 \rangle_F$, this work	0.6	0.552	0.018	-0.073	-0.185	0.373
$\langle r^2 \rangle_F$[9]	-	-	0.0433	-0.119	-0.187	0.378