

**Investigation of disordered state in the distribution of diluted
interaction coupling Using Ising Spin model in Two Dimension**

Abstract

In this study long range interaction coupling Ising spin-glass systems with $(J(r) \sim r^{-\gamma})$ are considered. Monte Carlo approach of the critical behavior is presented and also in presence of diverging thermodynamical fluctuations at the critical point the behavior of the model is shown to be of the Replica Symmetry Breaking type. The parallel tempering algorithm has been used to simulate the dynamical approach to equilibrium of such systems. The author has obtained phase transition temperatures for different linear lattice sizes ($L=10, 20, 30, 40$ & $\gamma = 2.0, 3.0$) but for $\gamma = 4.0$ and larger sizes there is no phase transition.

Key words: Long range interactions, Ising spin glass model, Monte Carlo Methods, Diluted bonds

1. Introduction

For long-range (LR) systems one can consider that their lower critical dimension is lower than the one of the corresponding short-range (SR) system. It is possible to think about there is a phase transition even in lower (one) dimensional system, if and only if the range of interaction is large enough. Lower dimensional models with distance dependent interactions actually allow to explore both LR and SR regimes by changing the power of distance between two spin sites and it is appropriate to compare the ordered phase in and out of the range of validity of the mean field approximation. Such a kind of approach is very useful for spin glass models that are known to have a rather very complicated states in the low-temperature phase in mean field theory [1,2]. Whether this structure exists in

finite-dimensional models with short-range interactions is, though, still a matter of debate. Theories alternative to the mean-field one have been proposed [3, 4], but SR systems are very tough to study analytically. Numerical simulations have been, thus, extensively employed, though with no conclusive indication on the nature of the spin-glass (SG) phase in finite dimension nor on the existence of a thermodynamic SG phase in presence of an external magnetic field. One-dimensional spin-glass model with power-law decaying interactions [5–9] makes probes on larger linear sizes possible. Moreover, if bonds are diluted [10–13] the run time in numerical simulations grows simply as the size L of the system, rather than proportionally to L^2 , as in fully connected systems. This is a fundamental issue because finite volume effects are strong in these models, so much that the very existence of the transition can be rather difficult to establish with canonical methods, e.g., when the interaction has a rapid decay and/or an external magnetic field is applied. In this study the author has considered a simulation for the overlap distribution function for $(y) = 3, 4$. For these two cases we found tangible evidence for the existence of second order phase transition from the paramagnetic to the spin glass phase. But for $y = 4$, double peaked distributions, typical of the spin glass phase are displayed at low T for small sizes. These results are in agreement with previous studies.

2. Models

For this work the model is Edwards-Anderson spin glass model[14-16] and this is based on a set of Ising spins $s_i = \pm 1$. The governing dynamics is

$$H = \sum_{(ij)} J_{ij} s_i s_j. \quad (2.1)$$

Where (ij) indicates a sum over nearest neighbors. The bonds (J_{ij}) are independent random variables drawn from a given distribution with mean zero and variance one. The theoreticians in this area, understand the spin-glass state

based on the overlap order parameter [14] which is defined as the overlap order parameter (as the hamming distance of the spin configurations of the replicas)[14]. For a given instance J of the quenched disorder the overlap is defined as:

$$q_J^{\alpha\beta} = \frac{1}{N} \sum_{i=1}^N \langle \sigma_i^{(1)} \rangle_{\alpha} \langle \sigma_i^{(2)} \rangle_{\beta} \quad (2.2)$$

Where $\langle \dots \rangle_{\alpha, \beta}$ is thermal average in the state α, β . Here N is the number of lattice sites. The numbers 1, 2 in the eq.(2.2) referee replicas 1 and 2.

3. Distance dependent interaction coupling

Previous studies showed that there is no phase transition for Ising model in two dimensions [18]. In order to show a phase transition we explore distance dependent interaction coupling. The Hamiltonian:

$$H = \sum_{(i < j)} J_{ij} s_i s_j. \quad (3.1)$$

The coupling constant is a quenched random variable whose probability of being non- zero decays with distance between two spin sites as[19]

$$p(J_{ij}) \propto r_{ij}^{-\gamma} \quad (3.2)$$

for $r_{ij} \gg 1$. Where r_{ij} is the distance between spins at the sites i and j. The probability of the non zero interaction coupling is equal and the coupling constant distribution is Gaussian with zero mean and variance

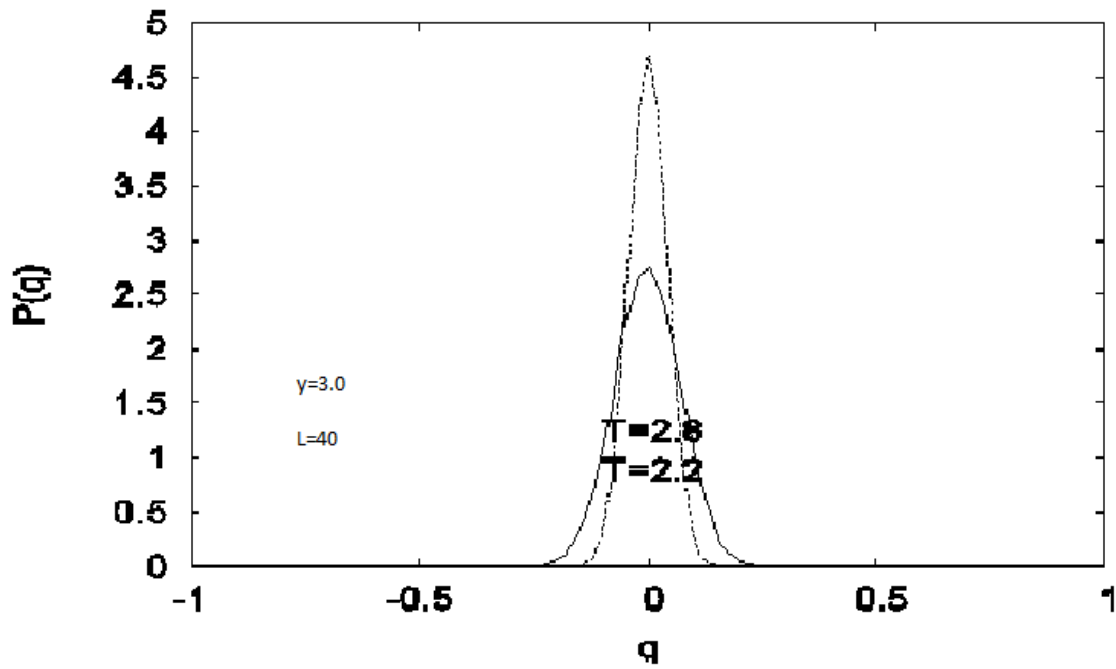
$(\overline{J_{ij}^2} \propto r_{ij}^{-\gamma})$ [20-22]. This approach allows us to study long and short range regimes by changing the power or (γ) [19] Because of each variable interacts with all the others, numerical simulations are very computer demanding, and it is hard to get clear experimental evidence supporting a specific spin-glass theory [20-21]. Taking in to consideration of this, introduce a diluted version of the model, where the mean coordination number is fixed [23] i.e. for this work we use the coordination number ($z = 6$).

4. Numerical Technique

The parallel tempering is an interesting and powerful twist of the simulating tempering (ST) first proposed by Charles Gayer in a conference proceeding (Gayer 1991). Just to implement a parallel tempering algorithm [17] one needs to make M replicas of the system (ensemble) to be analyzed, each of which characterized by different temperatures $T_1 > T_2 > \dots > T_M$. The main idea of this method is to simulate independently the standard Monte Carlo[24]for each copy, all of them at different temperatures and to swap periodically the configurations of two randomly chosen temperatures[25-26]so that it reduce the relaxation time of the simulation at each valley.

5. Results and Discussion

The plots show the existence of phase transition from paramagnetic to spin glass state for larger system sizes. Basically, in this case the researches considered overlap order parameter in order to identify spin glass phase. As depicted from the figures below we have shown the existence of spin glass phase for 2D Ising Spin model and for different sizes of lattices and power of distance between lattices. In Figs. 1, 2 and 3, the authors look at the probability distribution, averaged over disorder (100 & 200 depending on their size), of the overlap order parameter for $L = 40$, $y = 3.0, 4.0$, respectively. We used this comparison, in order to identify the effect of lattice size (L) and (y) on the spin glass state. For this purpose we have simulated two replicas of the system independently at each temperature.



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101 **Fig.1.** The plot of Probability distribution of overlap parameter ($P(q)$) versus order
 102 parameter (q) for different temperatures $T(\text{in K}) = 2.2; 2.8$ and $y = 3.0$, $L = 40$. This
 103 plot illustrates how pure paramagnetic phase at high temperature is translated to
 104 spin glass phase at low temperature. When the temperature is becoming lower and
 105 lower the **peak** is translated from single to double **peak** at some critical
 106 temperature. This temperature **(1.7k)** is phase transition temperature (from
 107 paramagnetic to spin glass phase). This figure is relatively for high temperature,
 108 and if we reduce the temperature the width of the pick is becoming wider and
 109 wider at some point we can observe double pick.

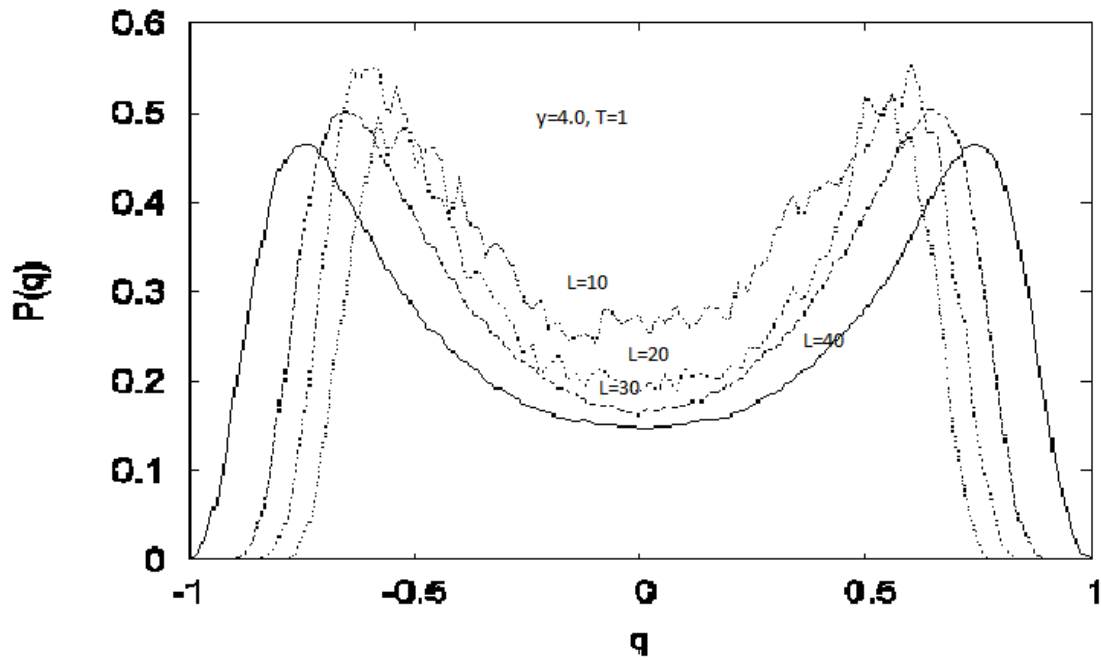
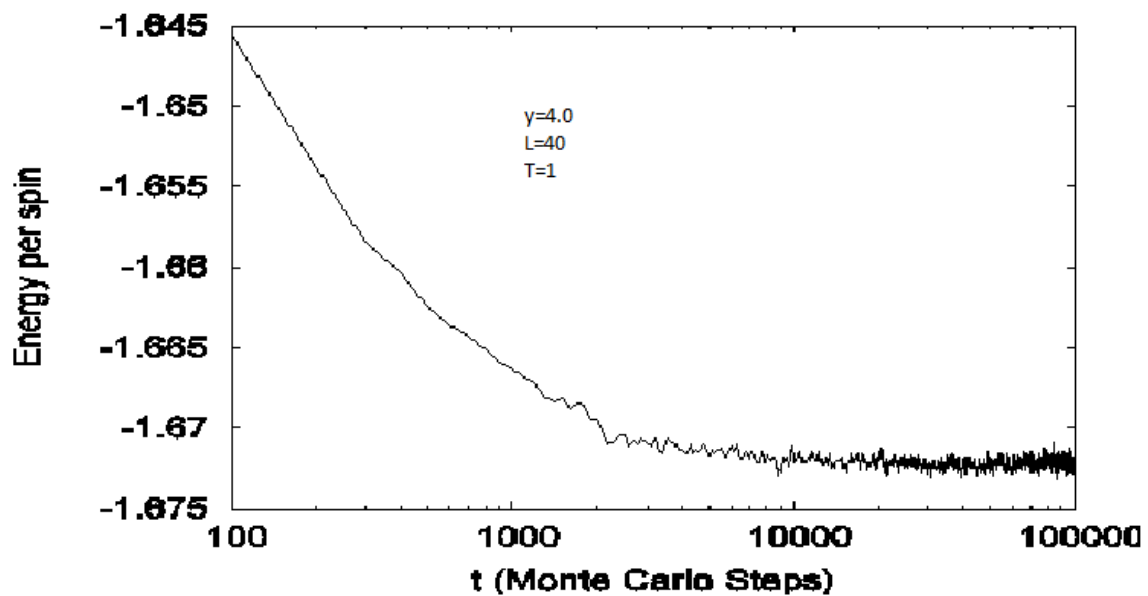


Fig.2 describes double peaked distributions, typical of the spin glass phase are displayed at low T for small sizes. However, the T_c at which the conversion from single peak in to double peak occurs, appear to continuously decrease with L . As one can see from fig.1, when temperature is becoming lower and lower, the size of the peak is also becoming wider and wider. At some critical temperature (transition temperature) the peak shifts from single to double. The figure is for $\gamma = 3.0$ and lattice sizes ($L=10,20,30,40$)



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120 **Figure3:** Schematic representation of Energy per spin versus t(Monte Carlo steps)
 121 for L=40 and T=1k, in this case we can see that how the energy demand at a
 122 standard Monte Carlo algorithm is high. As we can see from the plot for higher
 123 temperatures the thermalization time is also becoming higher. The thermalization
 124 time means in this case the time that the simulation gets more or less stable energy
 125 state. But if one keep increase the temperature the probability of getting this
 126 disorder state is null so that the time is lengthy.

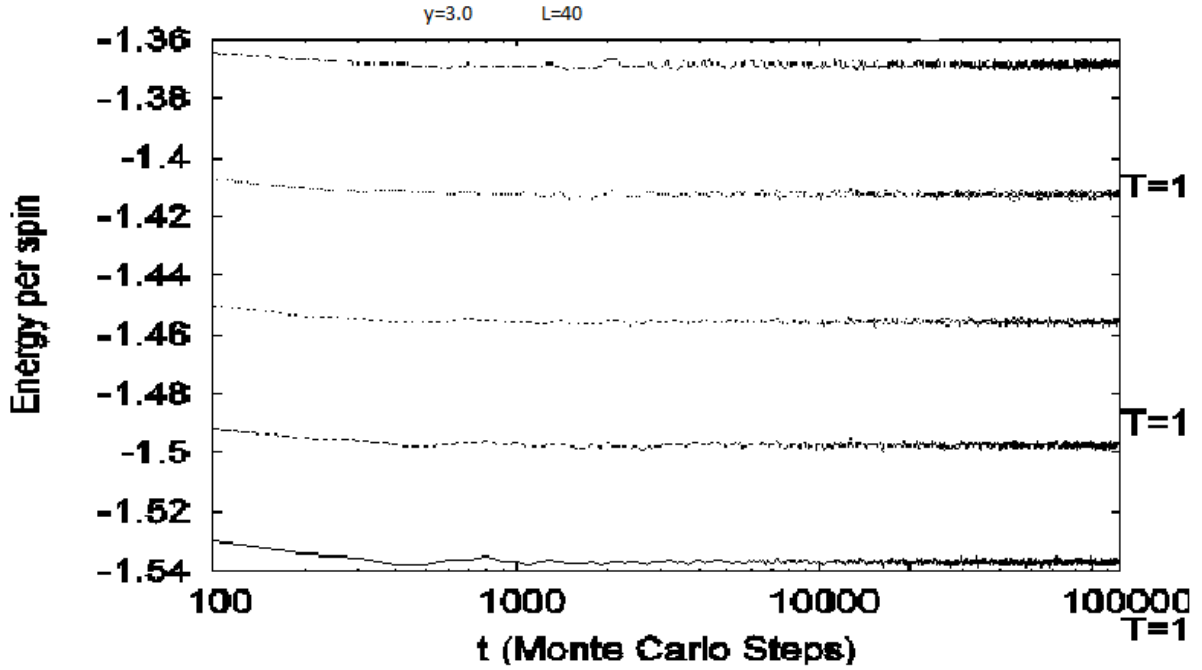


Figure 4: Schematic representation of Energy per spin versus t (Monte Carlo steps) for $y = 3.0$, $L = 40$, $T = 1.0k$. This figure is presented to indicate energy per spin versus Monte Carlo steps for the parameters mentioned above. In this case the system is thermalized but as ρ increases the existence of interaction coupling between distant spins is decreasing which reduces the simulation time.

6. Conclusion

Some of the physical quantities that we used diverge at the critical point. In order to avoid this complication in the simulation time we introduced finite size scaling approach. As usual for each realization of the bonds, two copies of the system are studied with different initial values of the spins and different random numbers for generating the spin flips of particular importance is the overlap between the two copies. We have taken a simulation for the probability of overlap distribution function for $y = 3, 4$. In both cases we found evidence for the existence of a phase transition from the paramagnetic to the spin glass phase. Specially for $y = 4$,

double peaked distributions, typical of the spin glass phase are displayed at low T for small sizes. These results are in agreement with previous studies.

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