

Magneto-Thermal Instability of Rotating Partially Ionized Hall Plasma Flowing Through Porous Medium

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Authors' contributions

This work was carried out in collaboration between all authors. " 'Author R. K.P.' designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. 'Author V. S.' managed the analyses of the study. 'Author A. K. P' managed the literature searches. All authors read and approved the final manuscript."

ABSTRACT

The magneto-thermal instability of an infinite homogeneous self-gravitating rotating partially ionized Hall plasma in the presence of viscosity, electrical resistivity, permeability, porosity, rotation and finite electron inertia is studied by means of linear perturbation analysis. A general dispersion relation is obtained using the normal mode analysis. Furthermore, the wave propagation parallel and perpendicular to the direction on magnetic field has been discussed. The stability of the system is discussed by applying Routh-Hurwitz criterion. For longitudinal propagation, it is found that the condition of radiative instability is independent of the magnetic field, collision frequency of neutrals with ions, Hall currents, finite electron inertia, porosity and viscosity; but for the transverse mode of propagation it depends on the strength of the magnetic field, rotation, porosity and electron inertia but independent of viscosity, permeability, electrical resistivity and collision frequency. From figures, we found that the effect of collision with neutrals, rotation, magnetic field and temperature dependent heat-loss function have a stabilizing influence while thermal conductivity and density dependent heat-loss functions have destabilizing influence on the self-gravitational instability of partially-ionized gaseous plasma. In addition, the classical Jeans condition regarding the rise of initial break up has been considerably modified due to the radiative heat-loss function.

Keywords: Thermal instability, Partially ionized plasma, Rotation, Self-gravitation, and Hall current.

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NOMENCLATURES

A	- No physical definition
B	- No physical definition
b	- Ratio of neutral and fluid densities
c	- Velocity of light [ms^{-1}]
c_p	- Specific heat of gas at constant pressure
S_a	- Adiabatic velocity of sound [ms^{-1}]
S_i	- Isothermal velocity of sound [ms^{-1}]
α	- No physical definition
G	- Universal gravitational constant [$\text{NM}^2\text{kg}^{-2}$]
H	- Strength of magnetic field in z direction [Am^{-1}]
\mathbf{H}	- Magnetic Field [Am^{-1}]
\mathbf{h}	- Perturbed magnetic field
h_x	- Perturbation of Magnetic field in x direction
h_y	- Perturbation of Magnetic field in y direction
h_z	- Perturbation of Magnetic field in z direction
i	- $\text{iota } (-1)^{1/2}$
k	- Wave number [m^{-1}]
k_x	- Wave number in x direction [m^{-1}]
k_y	- Wave number in y direction [m^{-1}]
\mathcal{L}	- Radiative heat-loss function [$\text{kg m}^{-3} \text{K}$]
\mathcal{L}_ρ	- Derivative of density- dependent heat-loss function
\mathcal{L}_T	- Derivative of temperature- dependent heat-loss function
P_0	- Initial fluid pressure
s	- Condensation of the medium
T_0	- Initial Temperature
t	- Time
ψ	- Gravitational Potential
V	- Alfvén velocity [ms^{-1}]
x	- x direction
z	- z direction
γ	- Ratio of specific heat
Δ_i	- Diagonal minors of Hurwitz matrices
ε	- Porosity of the medium
η	- Electrical resistivity of the medium
λ	- Thermal conductivity of the medium
ν	- Kinematic viscosity of fluid
ν_c	- Collision frequency
\mathbf{u}	- Velocity of fluid
\mathbf{u}_n	- Velocity of neutral particle
u	- Perturbation of fluid velocity in x direction
v	- Perturbation of fluid velocity in y direction
w	- Perturbation of fluid velocity in z direction
ρ_0	- Initial density of ionized component
ρ_n	- Density of neutral components
σ	- Frequency of harmonic disturbance
ω_{pe}	- Electron plasma frequency.
Ω_l	- No physical definition
Ω_j	- No physical definition

Ω_m	-	No physical definition
Ω_T	-	No physical definition
Ω_v	-	No physical definition
Ω_x	-	Component of rotation in x direction
Ω_z	-	Component of rotation in z direction
Ω	-	Rotation

1. INTRODUCTION

The problem of magneto-gravitational instability of interstellar matter is of considerable importance in connection with protostar and star formation in magnetic dust clouds. By considering plane wave perturbations to an infinite uniform medium, Jeans [1] has obtained that the disturbances would grow if their wave length exceeded a certain minimum λ_j is given by $\lambda_j^2 = \pi S^2 / G\rho$, where S , G , and ρ denote the sound velocity, gravitational constant and density of the medium, respectively. Comprehensive investigations of the Jens instability in self-gravitating fluids and plasmas are contained in Chandrashekar [2]. Owing to its relevance with protostar and star formation in magnetic dust clouds, it has attracted a wide attention in recent years. In this connection, many researchers [3-6] have discussed the problem of self-gravitational instability of plasma with different physical parameters such as viscosity, finite electrical conductivity, thermal conductivity, magnetic field and rotation.

In this direction, partially-ionized plasma represents a state which often exists in the universe. The interaction between the natural and the ionized gas components becomes importance in the cosmology. Kumar *et al.* [7] investigated the problem gravitational instability of an infinite homogeneous self-gravitating, rotating medium carrying uniform magnetic field in the presence of Hall Effect. Recently, the importance of influence of neutral-ion collision on the ionization rate in the solar photosphere, chromosphere and in cool interstellar cloud has pointed by Mamun and Shukla [8]. Jacobs and Shukla [9] have investigated the Jeans instability of partially ionized plasma under the effect of magnetic field. Borah and Sen [10] have studied the problem of gravitational instability of partially ionized plasma considering the effects of ions, electrons and charged dust grains.

Along with this, the effects of a Hall current and electrical conductivity are important to understand the problems of magnetic reconnection and break-down of the frozen-in condition in interstellar dynamics and in several other astrophysical situations. Ali and Bhatia [11] leading to the conclusion that the Hall currents are destabilizing in nature. Recently, the effects of Hall current and electrical resistivity on rotating self-gravitating anisotropic pressure plasma using generalized polytrope laws have been explored by Prajapati *et al.* [12]. Shaikh *et al.* [13, 14] have examined the effects of Hall currents, finite conductivity, and viscosity on self-gravitational instability of thermally conducting partially ionized plasma in a variable magnetic field.

In the past few years, it has been argued that thermal instability may be a reasonably good candidate, which can accelerate condensation, giving rise to localized structure which grows in density by losing heat, mainly through radiation. The first comprehensive analysis of thermal instability in a diffuse interstellar gas is first given by Field [15]. Bora and Talwar [16] have discussed the magneto-thermal instability of self-gravitating plasma with generalized ohm's law. Talwar and Bora [17] have analyzed the stability of a self-gravitating composite system of optically thin radiating plasma and stars. For cold stars, they have found that the thermal properties of the plasma have no effect on the situation and the system remains

unstable with respect to at least one stellar mode. The concept of radiation transfer was suggested as a necessary element for the understanding of processes taking place in stars Trintsadze *et al.* [18]. The linear thermal stability of a medium, subject to cooling, self-gravity and thermal conduction studied by Gomez-pelaez and Moreno-insertis [19]. The effect of dust particles on the thermal instability of an expanding plasma in presence of equilibrium cooling analyzed by Bora and Baruah [20]. Prajapati *et al.* [21] have discussed the problem of self-gravitational instability of rotating viscous Hall plasma with arbitrary radiative heat-loss functions and electron inertia. Recently, Kaothekar and Chhajlani [22] investigated the problem of self-gravitational instability of partially ionized plasma with radiative effects. The effect of radiation and electron inertia on the Jeans instability of partially ionized plasma have been studied by Dangarh *et al.* [23] and concluded that the Jeans criterion of instability is modifies to radiative instability criterion due to radiative heat-loss function. The effect of radiative heat-loss functions and finite ion Larmor radius (FLR) corrections on the gravitational instability of infinite homogeneous viscous plasma has been investigated by Kaothekar and Chhajlani [24]. Patidar *et al.* [25], in recent study, consider the problem of radiative instability of rotating two component plasma under the effect of electron inertia but does not include the effect of Hall current and permeability.

In all the above cited examples, we find that none of the authors have considered the combined effects of neutral-ion collision, rotation, Hall current, permeability, porosity, finite electron inertia and self-gravitation on the magneto-thermal instability of finitely electrically conducting, viscous partially ionized plasma flowing through a porous medium. Thus, in the present analysis, our aim is to analyze the magneto-thermal instability for a plasma model endowed with several mechanism namely Hall current, electrical resistivity, rotation, finite electron inertia, neutral-ion collision frequency, ion viscosity, permeability, porosity, and self-gravitational. This work is applicable to understand the phenomenon of small structure formation, magnetic reconnection phenomenon in space plasma.

2. LINEARIZED PERTURBATION EQUATIONS AND DISPERSION RELATION

We assume that the two components of the partially ionized plasma (the ionized fluid and the neutral gas) behave like a continuum fluid and their state velocities are equal. The effects of a magnetic field, field of gravity, and the pressure on the neutral components are neglected. Also it is assumed that the frictional force of the neutral gas on the ionized fluid is of the same order as the pressure gradient of the ionized fluid. Thus, we are considering only the mutual frictional effects between the neutral gas and the ionized fluid. It is assumed that the infinite homogeneous plasma medium is embedded in uniform magnetic field $\mathbf{H}(0,0,H)$.

The standard linearization process is applied to linearize the basic MHD Set of equations of the problem. We suppose all the physical quantities are the sum of their equilibrium and perturbed parts i.e. $u = u_0 + u'(\vec{r}, t)$, $H = H + h(\vec{r}, t)$, $\psi = \psi_0 + \psi'(\vec{r}, t)$, $\rho = \rho_0 + \rho'(\vec{r}, t)$, $p = p_0 + p'(\vec{r}, t)$, $T = T_0 + T'(\vec{r}, t)$, $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'(\vec{r}, t)$. It is considered that the fluid motion is steady with $u_0 = 0$. Thus the linearized perturbation equations governing the motion of hydro-magnetic thermally conducting two component plasma, rotating with a uniform angular velocity are given by

$$\frac{\partial \mathbf{u}'}{\partial t} = -\frac{1}{\rho_0} \nabla p' + \nabla \psi' + \frac{1}{4\pi\rho_0} (\nabla \times \mathbf{h}) \times \mathbf{H} + \frac{\rho_n}{\rho_0} \nu_c (\mathbf{u}'_n - \mathbf{u}') + \nu \left(\nabla^2 \mathbf{u}' - \frac{\mathbf{u}'}{K_1} \right) + 2(\mathbf{u}' \times \boldsymbol{\Omega}), \quad (1)$$

$$\frac{\partial \mathbf{u}'_n}{\partial t} = -\nu_c(\mathbf{u}'_n - \mathbf{u}'), \quad (2)$$

$$\varepsilon \frac{\partial \rho'}{\partial t} = -\rho_0 \nabla \cdot \mathbf{u}', \quad (3)$$

$$\nabla^2 \psi' = -4\pi G \rho', \quad (4)$$

$$\frac{1}{(\gamma-1)} \frac{\partial \rho'}{\partial t} - \frac{\gamma}{(\gamma-1)} \frac{p_0}{\rho_0} \frac{\partial \rho'}{\partial t} + \rho_0 (\mathcal{L}_\rho \rho' + \mathcal{L}_T T') - \lambda \nabla^2 T' = 0, \quad (5)$$

$$\frac{p'}{p_0} = \frac{T'}{T_0} + \frac{\rho'}{\rho_0}, \quad (6)$$

$$\frac{\partial \mathbf{h}}{\partial t} = \nabla \times (\mathbf{u}' \times \mathbf{H}) + \eta \nabla^2 \mathbf{h} - \frac{c}{4\pi N e} [\nabla \times \{(\nabla \times \mathbf{h}) \times \mathbf{H}\}] + \frac{c^2}{\omega_{pe}^2} \frac{\partial}{\partial t} \nabla^2 \mathbf{h}, \quad (7)$$

$$\nabla \cdot \mathbf{h} = 0. \quad (8)$$

where the symbols, G denotes gravitational constant, ν is kinematic viscosity, ν_c is the collision frequency between two components, K_1 is the medium permeability, λ is the coefficient of thermal conductivity, η is electrical resistivity, γ is adiabatic index, Ω ($\Omega_x, 0, \Omega_z$) is rotational frequency, c is velocity of light, N is number density of electron, e is the charge of electron, and u_n denote velocity of neutral particles. Here in equation (5), $\mathcal{L}_{\rho,T}$ denote the partial derivatives of density dependent $(\partial \mathcal{L} / \partial \rho)_T$ and temperature dependent $(\partial \mathcal{L} / \partial T)_\rho$ heat-loss functions respectively.

We now solve equation (1) to (8) using normal mode analysis with assumption, that all the perturbed quantity vary as

$$\exp. \{i(k_x x + k_z z) + \sigma t\}. \quad (9)$$

where σ is the frequency of harmonic disturbances, $k_{x,z}$ are the wave numbers in transvers and longitudinal directions to the magnetic field, such that $k_x^2 + k_z^2 = k^2$. eqn. (5) and eqn. (6) yield the following relation between δp and $\delta \rho$:

$$\delta p = \left(\frac{A + \sigma S_a^2}{\sigma + B} \right) \delta \rho, \quad (10)$$

where $S_a = \sqrt{\gamma p_0 / \rho_0}$ is the adiabatic velocity of sound in the medium and the parameters A and B are defined by

$$A = (\gamma - 1) \left(\mathcal{L}_T T_0 - \mathcal{L}_\rho \rho_0 + \frac{\lambda k^2 T_0}{\rho_0} \right),$$

$$B = (\gamma - 1) \left(\frac{\mathcal{L}_T T_0 \rho_0}{p_0} + \frac{\lambda k^2 T_0}{p_0} \right). \quad (11)$$

We obtain the four linear equations in terms of the amplitude components u, v, w, s as

$$\left(M + \frac{k^2 V^2 a}{D}\right)u - (Q_1 + 2\Omega_z)v + \frac{ik_x}{k^2}\Omega_T^2 s = 0. \quad (12)$$

$$(Q_1 + 2\Omega_z)u + \left(M + \frac{k_z^2 V^2 a}{D}\right)v - 2\Omega_x w = 0. \quad (13)$$

$$2\Omega_x v + Mw + \frac{ik_z}{k^2}\Omega_T^2 s = 0. \quad (14)$$

$$\left(\frac{ik_x k^2 V^2 a}{D}\right)u - (ik_x Q_1 + 2ik_x \Omega_z - 2ik_z \Omega_x)v - (\sigma \varepsilon M + \Omega_T^2)s = 0. \quad (15)$$

where $s = (\rho'/\rho_0)$, is the condensation of the medium, $V = (H/\sqrt{4\pi\rho_0})$ is the Alfven velocity. Also we have assumed the following substitutions to avoid the complexity in algebraic calculations

$$\begin{aligned} N_1 &= \left(M + \frac{k^2 V^2 a}{D}\right), \quad N_2 = \left(M + \frac{k_z^2 V^2 a}{D}\right), \quad N_3 = (\varepsilon \sigma M + \Omega_T^2), \\ M &= \left(\sigma + \frac{\sigma b v_c}{\sigma + v_c} + \Omega_v\right), \quad D = (a^2 + k_z^2 Q^2 k^2), \quad Q = \left(\frac{cH}{4\pi Ne}\right), \quad a = (\alpha \sigma + \Omega_m), \\ b &= \left(\frac{\rho_n}{\rho}\right), \quad \Omega_m = (\eta k^2), \quad \Omega_v = v \left(k^2 + \frac{1}{k_1}\right), \quad \Omega_T^2 = \left(\frac{\sigma \Omega_j^2 + \Omega_I^2}{\sigma + B}\right), \\ \Omega_I^2 &= (k^2 A - 4\pi G \rho_0 B), \quad \Omega_j^2 = (k^2 S_a^2 - 4\pi G \rho_0), \quad Q_1 = \left(\frac{k_z^2 Q V^2 k^2}{D}\right). \end{aligned}$$

Now we can write equation (12)-(15) in the matrix form, to obtain the dispersion relation, as

$$[X] [Y] = 0. \quad (16)$$

Here $[X]$ is the fourth order matrix and $[Y]$ is the single column matrix of elements $[u, v, w, s]$. The vanishing of $[X]$ gives the following equation,

$$\begin{aligned} &-N_3 \left[N_1 (N_2 M + 4\Omega_x^2) + M (Q_1 + 2\Omega_z)^2 \right] + \frac{k_x^2 \Omega_T^2}{k^2} \left[4M \Omega_z^2 + \frac{a M N_2 k^2 V^2}{D} + \right. \\ &4ak2V2\Omega_x2D + MQ12 + 4\Omega_z Q1M + kz2 \\ &k24\Omega_x2\Omega_T2N1 - 2\Omega_x kxkz\Omega_T22\Omega_zk2 + V2kz2QDN1 + M - Q1k2V2D - 2\Omega_z aV2 \\ &\left. D = 0 \right]. \end{aligned} \quad (17)$$

The dispersion relation (17) shows a general dispersion relation for wave propagation in an homogeneous self-gravitating partially ionized plasma incorporating the effects of magnetic field, rotation, thermal conductivity, radiative heat-loss function, Hall current, electrical resistivity, permeability, finite electron inertia, viscosity and porosity of the medium. We find that in (17) the terms due to the Hall current have entered through the factor Q .

The preceding dispersion relation (17) is the modified form of the dispersion relation obtained by Dangarh *et al.* [23] due the consideration of Hall current, rotation, viscosity,

permeability, porosity, and electrical resistivity, excluding electron inertia. Also ignoring the effect of rotation, Hall current and neutral particles the above dispersion relation reduces to Kaothekar and Chhajlani [24] excluding permeability and FLR correction. Again equation (24) gives the same result as Bora and Talwar [16] by ignoring rotation permeability and neutral ion collision in our case. Now, we will reduce (17) in two different modes of propagation, parallel and perpendicular to the magnetic field to investigate the effects of considered parameter, separately and simultaneously.

3. ANALYSIS OF THE DISPERSION RELATION

3.1 Longitudinal Mode of Propagation ($k \parallel H$)

For this case we assume all the perturbations longitudinal to the direction of the magnetic field i.e. ($k_z = k, k_x = 0$). This is the dispersion relation reduces in the simple form to give

$$-N_1 N_2 N_3 M - 4N_1 N_3 \Omega_x^2 - M N_3 \left(\frac{Q V^2 k^4}{D} + 2\Omega_z \right)^2 + 4N_1 \Omega_x^2 \Omega_T^2 = 0. \quad (18)$$

Equation (18) gives the general dispersion relation for an infinite homogeneous, uniformly magnetized, self-gravitating, rotating, partially ionized Hall plasma having finite electrical and thermal conductivity, porosity, permeability, viscosity, and radiative heat-loss functions when the disturbances are propagating parallel to the magnetic field. Again for simplicity, the dispersion relation (18) is discussed for axis of rotation is along and perpendicular to the magnetic field separately.

3.2 Axis of Rotation along Magnetic Field

When the axis of rotation is along the magnetic field, we put $\Omega_x = 0$ and $\Omega_z = \Omega$, the dispersion relation (18) reduces to

$$-N_3 M \left(N_1 N_2 + 4\Omega^2 + \frac{k^4 Q^2 V^4 k^4}{D^2} + 4\Omega \frac{Q V^2 k^4}{D} \right) = 0. \quad (19)$$

This dispersion relation (19) represents wave propagation, in longitudinal direction with axis of rotation is parallel to magnetic field, for self-gravitating partially ionized plasma under influence of Hall effect, radiative effect, electrical conductivity, electron inertia, viscosity, and permeability of porous medium. Dispersion relation (19), on substituting the values of N_1, N_2, N_3, M , and D , has three different independent modes of propagation corresponding to equations.

$$\sigma^2 + \sigma P + \Omega_v \nu_c = 0. \quad (20)$$

$$\sigma^8 + \sigma^7 \alpha_1 + \sigma^6 \alpha_2 + \sigma^5 \alpha_3 + \sigma^4 \alpha_4 + \sigma^3 \alpha_5 + \sigma^2 \alpha_6 + \sigma \alpha_7 + \alpha_8 = 0. \quad (21)$$

$$\sigma^4 + \sigma^3 (P + B) + \sigma^2 \left[\nu_c \{ \Omega_v + (1 + b) B \} + \frac{\Omega_j^2}{\varepsilon} + B \Omega_v \right] + \sigma \left[\nu_c \left\{ \frac{\Omega_j^2}{\varepsilon} + B \Omega_v \right\} + \Omega I 2 \varepsilon + \nu_c \Omega I 2 \varepsilon \right] = 0. \quad (22)$$

[Here $P = \{v_c(1 + b) + \Omega_v\}$. and α_1 to α_8 in appendix A]

The first of these, equation (20) is identical to Dangarh *et al.* [23] when the contribution viscosity and permeability is ignored and also similar to Patidar *et al.* [25] for non permeable medium.

Equation (20) does not admit a positive real root or complex root whose real part is positive, meaning thereby that the system is stable. Therefore (20) represents the stable damped mode due to viscosity of medium, modified by the effect of collision frequency and permeability. Hence, we can conclude that the viscous partially ionized fluid is more stable than viscous fluid.

The second one, equation (21) represents Alfvén mode of propagation coupled with the effects of Hall current, collision frequency, viscosity, permeability, rotation, electrical resistivity, and electron inertia. Equation (21) is the same as obtained by Patidar *et al.* [25] when ignoring the effect of Hall current and permeability. Also (21) is the modified form of dispersion relation of Prajapati *et al.* [21] due to the effect of neutral-ion collision and porosity of the medium. Again the dispersion relation of Kothekar and Chhajlani [22] can be modify in form of (21) by considering the effect of Hall current, electron inertia, rotation, porosity and permeability of porous medium in their case.

For perfectly electrically conducting medium [$\Omega_m = 0$] and in the absence of collision frequency and Hall current [$v_c = Q = 0$] the dispersion relation (21) reduces to

$$\sigma^4 + \sigma^3 2\Omega_v + \sigma^2 \left[\Omega_v^2 + \frac{2V^2 k^2}{\alpha} + 4\Omega^2 \right] + \sigma \left[\frac{2\Omega_v V^2 k^2}{\alpha} \right] + \frac{V^4 k^4}{\alpha^2} = 0. \quad (23)$$

The stability of the system, represented by preceding equation, is discussed using Routh-Hurwitz criterion, since all the coefficients of equation (23) are positive that the necessary condition for instability of the system is satisfied. To obtain the sufficient condition, the principal diagonal minors of Hurwitz matrix must be positive, which are shown below

$$\Delta_1 = 2\Omega_v > 0.$$

$$\Delta_2 = 2\Omega_v \left[\Omega_v^2 + 2\frac{V^2 k^2}{\alpha} + 4\Omega^2 \right] > 0.$$

$$\Delta_3 = 4\Omega_v^2 \frac{V^2 k^2}{\alpha} \left[\Omega_v^2 + 4\Omega^2 \right] > 0.$$

$$\Delta_4 = \frac{V^4 k^4}{\alpha^2} \Delta_3 > 0.$$

We see that all Δ 's are positive so we find that a magnetized, rotating, viscous plasma in perfectly electrically conducting medium is a stable system. For an in-viscid fluid, [$\Omega_v = 0$], (24) reduces to

$$\sigma^4 + \sigma^2 \left[\frac{2V^2 k^2}{\alpha} + 4\Omega^2 \right] + \frac{V^4 k^4}{\alpha^2} = 0. \quad (24)$$

It is evident from equation (24) that

$$\sigma_{1,2}^2 = - \left[\frac{V^2 k^2}{\alpha} + 2\Omega^2 \right] \pm 2\Omega \left(\frac{V^2 k^2}{\alpha} + \Omega^2 \right)^{1/2}. \quad (25)$$

Thus, we see in equation (25), the two Alfvén waves modified by electron inertia and rotation, moving in opposite direction. Now for in-viscid [$\Omega_v = 0$], finitely conducting medium in the absence of neutral particles [$v_c = 0$], eq. (21) can be written as

$$\sigma^2 \alpha^2 + \sigma 2\Omega_m \alpha + \Omega_m^2 + 2Q^2 k^4 = 0. \quad (26)$$

$$\sigma^4 \alpha^2 + \sigma^3 2\Omega_m \alpha + \sigma^2 [\Omega_m^2 + 2V^2 k^2 \alpha + Q^2 k^4 + 4\Omega^2 \alpha^2] + \sigma 2\Omega_m [V^2 k^2 + \alpha 4\Omega 2 + V 4 k 4 + 4\Omega 2\Omega m 2 + 4\Omega 2Q 2 k 4 + 4\Omega Q k 4 V 2] = 0. \quad (27)$$

Equation (26) represents the effect of Hall current and finite electrical resistivity. It may be remarked that due to resistivity and Hall current, mode of propagation is periodically in nature which is quenched by resistivity parameter as $\exp. (-\eta k^2)t$ and in the absence of Hall current this mode is damped stable mode due to electrical resistivity. Equation (27) is fourth degree equation having all its coefficients positive and the principle diagonal minors of Hurwitz's matrix are also positive hence this mode shows stability.

The last one, equation (22) represents combined effect of radiative heat-loss function, thermal conduction, and self-gravitation. It is evident from (22) that this mode is independent of the effects of a magnetic field, electrical resistivity, Hall current, rotation, and electron inertia. In the absence of neutral-ion collision, and porosity (22) reduces to Prajapati *et al.* [21] also (22) is similar to Patidar *et al.* [25] by ignoring the effects of permeability and Hall current.

The dispersion relation (22) is a fourth degree equation which may be reduced to particular cases so that the effect of each parameter is analyzed separately.

For thermally non-conducting, non-radiating, non viscous, self-gravitating fully ionized fluid we have [$A = B = \nu_c = \nu = 0$], and for non porous medium [$\varepsilon = 1$], the dispersion relation (22) reduces to

$$\sigma^2 + S_a^2 k^2 - 4\pi G \rho_0 = 0. \quad (28)$$

This is the same equation obtained by Jeans for gravitational instability of infinite homogeneous self-gravitating fluid. It is clear from equation (28) that when $\Omega_j^2 < 0$, the product of the roots of equation (28) must, therefore, be negative. This implies that at least one root of σ is positive. Hence, the system is unstable when

$$\Omega_j^2 = (S_a^2 k^2 - 4\pi G \rho_0) < 0.$$

$$k < k_j = \left(\frac{4\pi G \rho_0}{S_a^2} \right)^{1/2}. \quad (29)$$

where, k_j is the Jeans wave number. The fluid is unstable for all wave number $k < k_j$. It is evident from (29) that the presence of neutral particles does not alter the Jeans' criterion of instability.

For non-radiating but thermally conducting, viscous and self-gravitating fluid having neutral particles, (22) reduces to

$$\sigma^4 + \sigma^3(P + \Omega_k) + \sigma^2 \left[\Omega_v \nu_c + P \Omega_k + \frac{\Omega_{j1}^2}{\varepsilon} \right] + \sigma \left[\nu_c \left(\frac{\Omega_{j1}^2}{\varepsilon} + \Omega_v \Omega_k \right) + \frac{\Omega_k \Omega_{j1}^2}{\varepsilon} \right] + \frac{\nu_c \Omega_k \Omega_{j1}^2}{\varepsilon} = 0. \quad (30)$$

where $\Omega_k = (\lambda \gamma k^2 / \rho_0 c_p)$ and c_p is the specific heat of the gas at constant pressure, $\Omega_{j1}^2 = (S_i^2 k^2 - 4\pi G \rho)$, and $S_i = \sqrt{p/\rho}$, is the isothermal velocity of the sound. It is clear from the constant term of equation (30) that the system leads to instability if $\Omega_{j1}^2 < 0$, which gives $S_i^2 k^2 - 4\pi G \rho_0 < 0$, and corresponding wave number as

$$k < k_{j1} = \left(\frac{4\pi G \rho_0}{S_i^2} \right)^{1/2}. \quad (31)$$

where k_{j1} is the modified Jeans wave number for thermally conducting system. On comparing equation (29) and (31) we observe that the adiabatic sound velocity is replaced by isothermal one in the Jeans expression for thermally conducting medium. Again from (31) we can say that the collision between neutral and ionized component does not affect the Jeans expression for gravitational instability.

It is clear from equation (31) that, when $\gamma > 1$ then ($S_a > S_i$) therefore owing to thermal conduction Jeans wave number is increased as a result the critical wavelength is reduced. Thus the size of initial break up is reduced; the destabilizing effect is produced in the interstellar medium. If we consider non-gravitating but thermally conducting plasma incorporated with radiative heat-loss function then the expression for critical wave number is given as

$$k < k_{j2} = k_j \left(\frac{\gamma \mathcal{L}_T}{\mathcal{L}_T - \frac{\rho_0 \mathcal{L}_\rho}{T}} \right)^{1/2}. \quad (32)$$

Here k_{j2} is the modified critical wave number due to inclusion of radiative heat-loss function. Hence, for wave number $k < k_{j2}$, the system is unstable. It is clear from (32) that in this case the critical Jeans wave number depends on the derivatives of the heat-loss function with respect to local temperature and local density in the configuration. The critical Jeans wave number vanishes if the heat-loss function is independent of temperature ($\mathcal{L}_T = 0$) and $\sqrt{\gamma}$ times of original critical Jeans wave number if the heat-loss function is purely temperature-dependent ($\mathcal{L}_\rho = 0$). It may be remarked that the critical wave number decreases or increases as the heat-loss functions respectively increases or decreases with increases in density.

Owing to simultaneous effect of all the parameters represented by the original dispersion relation (22), the condition of instability obtained from eq. (22) form constant term is

$$\Omega_I^2 = \left[k^2 \left(T_0 \mathcal{L}_T - \rho_0 \mathcal{L}_\rho + \frac{\lambda k^2 T_0}{\rho_0} \right) - 4\pi G \rho_0 \left(\frac{T_0 \rho_0 \mathcal{L}_T}{p_0} + \frac{T_0 \lambda k^2}{p_0} \right) \right] < 0. \quad (33)$$

It is evident from (33) that the Jeans' criterion of instability is modified due to inclusion of thermal conductivity and radiative term. Also in other word we can say that, the condition of thermal instability obtained by Field [15] is modified due to self-gravitation. The inequality (33) is similar to that of obtained by Bora and Talwar [16] and also to that of Patidar *et al.* [28] and can be solved to get the following expression of critical Jeans wave number

$$k_{j3} = \frac{1}{2^{1/2}} \sqrt{\left[\left\{ \frac{4\pi G \rho_0}{S_i^2} + \frac{\rho_0^2 \mathcal{L}_\rho}{\lambda T_0} - \frac{\rho_0 \mathcal{L}_T}{\lambda} \right\} \pm \left\{ \left(\frac{4\pi G \rho_0}{S_i^2} + \frac{\rho_0^2 \mathcal{L}_\rho}{\lambda T_0} - \frac{\rho_0 \mathcal{L}_T}{\lambda} \right)^2 + \frac{16\pi G \rho_0^2 \mathcal{L}_T}{\lambda S_i^2} \right\}^{1/2} \right]}. \quad (34)$$

It may be noted here that modified critical Jeans wave number involves, derivatives of temperature dependent and density dependent heat-loss function and thermal conductivity of the medium. If we assume that the radiative heat-loss function is purely temperature dependent ($\mathcal{L}_\rho = 0$), increases with temperature ($\mathcal{L}_T > 0$) then (35) is reduces to (31) to obtain the condition of monotonic instability. However, if instead the arbitrary radiative heat-loss function decreases with temperature ($\mathcal{L}_T < 0$), the instability arises for k^2 lying between the values $(|\mathcal{L}_T|/\lambda)$ and $(4\pi G \rho_0/S_i^2)$ for parallel propagation. Furthermore, if it is considered that heat-loss function is purely density dependent ($\mathcal{L}_T = 0$) then the condition of instability is given as

$$k < k_{j4} = \left(\frac{4\pi G \rho}{S_i^2} + \frac{\rho^2 \mathcal{L}_\rho}{\lambda T} \right)^{1/2}. \quad (35)$$

It is evident from (35) that the critical wave number is increased or decreased, depending on whether the arbitrary radiative heat-loss function is an increasing or decreasing function of the density.

In order to discuss the dynamical stability of the system represented by (22), we applied the Routh-Hurwitz criterion. According to this criterion, the necessary condition is that all the coefficients of the polynomial equation (22) should be positive. In order to satisfy the sufficient condition, we calculate the minors of the Hurwitz matrix formed by these coefficients, which are

$$\Delta_1 = (P + B) > 0. \text{ as } \gamma > 1$$

$$\Delta_2 = \left[P\Omega_v \nu_c + B\nu_c \frac{\Omega_j^2}{\varepsilon} + \frac{\Omega_v \Omega_j^2}{\varepsilon} + \frac{B\Omega_j^2}{\varepsilon} + PB^2 + P^2B - \frac{\Omega_I^2}{\varepsilon} \right] > 0.$$

$$\Delta_3 = \left[PB\Omega_v^2 \nu_c^2 + \frac{\Omega_j^2 \nu_c^2 \Omega_v P}{\varepsilon} + \frac{P\Omega_v \nu_c \Omega_I^2}{\varepsilon} + P^2 \nu_c \Omega_v B^2 + \frac{P^2 \nu_c B \Omega_j^2}{\varepsilon} + \frac{P^2 \Omega_I^2 B}{\varepsilon} + \right. \\ \left. P\nu_c \Omega_v B^3 + \frac{\nu_c B^2 \Omega_j^2 P}{\varepsilon} + \frac{\nu_c \Omega_v B^2 \Omega_j^2}{\varepsilon} + \frac{\nu_c B \Omega_I^4}{\varepsilon} + \frac{\Omega_j^2 \Omega_I^2 B}{\varepsilon} + \frac{B\nu_c^2 \Omega_v \Omega_j^2}{\varepsilon} + \frac{\Omega_v^2 \nu_c B \Omega_j^2}{\varepsilon} + \right]$$

$$\left[\frac{\Omega_j^2 \Omega_I^2 B v_c}{\varepsilon^2} + \frac{\Omega_j^2 \Omega_I^2 \Omega_v}{\varepsilon^2} + \frac{v_c B \Omega_j^4}{\varepsilon} + \frac{v_c \Omega_v \Omega_j^4}{\varepsilon} + \frac{v_c B^2 \Omega_I^2}{\varepsilon} + \frac{\Omega_v B^2 \Omega_I^2}{\varepsilon} - \frac{\Omega_I^2}{\varepsilon} \left(\frac{v_c \Omega_j^2}{\varepsilon} + \frac{\Omega_I^2}{\varepsilon} + \Omega_v v_c B + P^2 v_c + 2P v_c B \right) \right] > 0.$$

$$\Delta_4 = \frac{v_c \Omega_I^2 \Delta_3}{\varepsilon} > 0.$$

Since these all Δ 's are positive, thereby, satisfying the Routh-Hurwitz criterion, according to which equation (22) will not include any positive real root of σ or a complex root whose real part is positive. Therefore the system represented by (22) will remain stable if $\Omega_I^2 = [k^2 A - 4\pi G \rho_0 B] > 0$. Thus we find that for longitudinal wave propagation the gravitating plasma is stable if the condition $[k^2 A > 4\pi G \rho_0 B]$ is satisfied.

3.3 Non-Gravitating Hydromagnetic Fluid

In this section, for non-gravitating hydromagnetic fluid, two modes of propagation are similar to as discussed in equations (20) and (21) but the third mode of propagation is quite different from that of discussed in (22) for self-gravitating fluid. The dispersion relation for non-gravitating viscous fluid subjected to general heat-loss function and Hall current with thermal and electrical conductivity flowing through porous medium is obtained from the third factor of equation (19) and given as

$$\sigma^4 \varepsilon + \sigma^3 (P + B) \varepsilon + \sigma^2 [v_c \{\Omega_v + (1 + b)B\} \varepsilon + S_a^2 k^2 + B \Omega_v \varepsilon] + \sigma [v_c S_a^2 k^2 + \varepsilon B \Omega_v v_c + k^2 A + v_c k^2 A] = 0. \quad (36)$$

Evidently, if $A < 0$ then above equation will possess at least one positive root implying thereby instability of the system. The condition of instability for non-gravitating hydromagnetic fluid is given as

$$\left(\mathcal{L}_T T_0 - \mathcal{L}_\rho \rho_0 + \frac{\lambda k^2 T_0}{\rho_0} \right) < 0.$$

The critical wave number is given as

$$k_{j3} = \sqrt{\frac{(\rho_0 \mathcal{L}_\rho - \mathcal{L}_T T_0) \rho_0}{\lambda T_0}}. \quad (37)$$

We notice the effect of neutral-ion collision does not affect the condition of instability but its presence modifies the dispersion relation (36) as well as the growth rate of instability of and non-gravitating radiating Hall plasma medium. Also we find that condition of instability (37) is independent of viscosity, Hall current, rotation, electrical resistivity, porosity and finite electron inertia and is identical to Field [15]. It is clear from equation (37) that when the arbitrary radiative heat-loss function is independent of temperature of the configuration (i.e. $\mathcal{L}_T = 0$), then

$$k_{j4} = \rho_0 \sqrt{\frac{\mathcal{L}_\rho}{\lambda T}}. \quad (38)$$

If inequalities (38) is applied for increases with temperature ($\mathcal{L}_T > 0$), then the condition of monotonic instability is given as $k < k_{j4}$. However, if instead the arbitrary radiative heat-loss function decreases with temperature ($\mathcal{L}_T < 0$), the instability arises for k^2 lying between the values $(|\mathcal{L}_T|/\lambda)$ and $(\rho_0 \mathcal{L}_p / \lambda T)$ for parallel propagation.

The mechanism underlying the thermal instability is a heat-loss function which decreases with temperature and increases with density and is important to analyze the instability phenomena of various astrophysical problems, such as coronal condensations. For the solar corona, the heat-loss function depends on local density and temperature so that the above conditions of thermal instability are satisfied, resulting in, the local temperature falls, the local pressure decreases leading to condensation of the cool plasma which radiates faster because of density rise.

3.4 Axis of Rotation Perpendicular to The Magnetic Field

In the case of a rotation axis perpendicular to the magnetic field we put $\Omega_x = \Omega$, and $\Omega_z = 0$ in the dispersion relation(18) and this gives.

$$-M \left[N_1 N_2 N_3 + N_3 \left(\frac{V^4 k^4 Q^2}{A^2} \right) - 4\sigma \Omega^2 N_1 \right] = 0. \quad (39)$$

This dispersion relation is the product of two independent factors. These factors show the mode of propagations incorporating different parameters as discussed below.

Dispersion relation (39) has two different independent modes of propagation corresponding to equations.

$$\sigma^2 + \sigma P + \Omega_v \nu_c = 0. \quad (40)$$

$$\sigma^{12} + \sigma^{11} C_1 + \sigma^{10} C_2 + \sigma^9 C_3 + \sigma^8 C_4 + \sigma^7 C_5 + \sigma^6 C_6 + \sigma^5 C_7 + \sigma^4 C_8 + \sigma^3 C_9 + \sigma^2 C_{10} + \sigma C_{11} + C_{12} = 0. \quad (41)$$

The first of these, (40) is similar to (20) and represents the combined stable effect of viscosity, permeability and neutral ion collision in damped oscillatory form.

The last one, (41) is, very lengthy and complex to write here, but to discuss the condition of instability we need the constant term of the last coefficients. Equation (41) represents the general dispersion relation for an infinite homogeneous, rotating, thermally conducting, self-gravitating, and viscous partially ionized plasma flowing through porous medium incorporating radiative heat-loss function and magnetic field, when the disturbances are propagating along the direction of magnetic field and the axis of rotation is perpendicular to the direction of magnetic field. The constant term of the last coefficient of (41) is given by

$$C_{12} = \nu_c^3 \Omega_I^2 \left[\frac{\Omega_v \Omega_m}{\alpha} \left(\frac{\Omega_v \Omega_m^3}{\alpha^3} + \frac{2\Omega_v \Omega_m Q^2 k^4}{\alpha^3} + \frac{2V^2 k^2 \Omega_m^2}{\alpha^3} + \frac{2V^2 k^2 Q^2 k^4}{\alpha^3} \right) + \right. \\ \left. Q4k8\alpha4\Omega_v2 + V4k4\Omega_m2\alpha4 + V4k8Q2\alpha4. \right]$$

The condition of instability is obtained from constant term of equation (41) and gives as

$$\Omega_I^2 = k^2 A - 4\pi G \rho_0 B < 0. \quad (42)$$

The above condition of instability is identical to the condition (33) for radiative instability. We find that the condition of instability for this mode of propagations, in both the cases of rotation parallel and perpendicular to a magnetic field is the same and there is no effect of the direction of rotation on the instability condition. Also we can conclude that the presence of finite electron inertia, Hall current porosity and neutral particles does not alter the condition of radiative instability in longitudinal mode of propagation, but presence of these parameters modifies the growth rate of instability.

3.5 Non-Gravitating Hydromagnetic Fluid

In this case, for non-gravitating hydromagnetic fluid, first mode of propagation is identical to equations (20) hence no need to be discussed here but the last factor is affected and for non gravitating hydromagnetic fluid the coefficient of the last term of (41) reduces to

$$\nu_c^3 \left[\frac{\Omega_v \Omega_m}{\alpha} \left(\frac{\Omega_v \Omega_m^3}{\alpha^3} + \frac{2\Omega_v \Omega_m Q^2 k^4}{\alpha^3} + \frac{2V^2 k^2 \Omega_m^2}{\alpha^3} + \frac{2V^2 k^2 Q^2 k^4}{\alpha^3} \right) + \left(\frac{Q^4 k^8}{\alpha^4} \Omega_v^2 + \frac{V^4 k^4 \Omega_m^2}{\alpha^4} + \right. \right. \\ \left. \left. V^4 k^8 Q^2 \alpha^4 k^2 A = 0. \right. \right. \quad (43)$$

The dispersion relation (43) shows the combined influence of viscosity, permeability of porous medium, rotation, Hall current, electrical and thermal conductivity on thermal instability of magnetized non-gravitating plasma.

Equation (43) can be converted to the equations of previous work of Field [15] and Ibanez [26], by ignoring the effect of finite electron inertia, viscosity, rotation, finite electrical resistivity and collision frequency between two components of partially ionized plasma and setting molecular weight unity in their cases.

Hence the present results are the modified results of Field [15] and Ibanez [26] with these considered parameters. The condition of instability of Field [15] and Ibanez [26] is modified due to our consideration of self gravitation of the medium. The growth rate of instability of this dispersion relation will also be modified due to presence of these parameters. If we ignore the effects of finite electron inertia, finite electrical resistivity, collision frequency and viscosity (43) reduces to the one similar to that obtained by Aggarwal and Talwar [27].

3.6 Transverse Mode of Propagation ($k \perp H$)

For this case we assume all the perturbations are propagating perpendicular to the direction of the magnetic field, for, our convenience, we take $k_x = k$, and $k_z = 0$, the general dispersion relation (17) reduces to

$$-\left\{ \sigma M^4 + \sigma M^3 \frac{V^2 k^2}{a} + M^3 \Omega_T^2 + \sigma 4M^2 \Omega_x^2 + 4\Omega_x^2 M \Omega_T^2 + \frac{V^2 k^2}{a} 4\sigma M \Omega_x^2 + \right. \\ \left. 4M^2 \Omega_z^2 \sigma = 0. \right. \quad (44)$$

We find that in the transverse mode of propagation the dispersion relation (44) is modified due to the presence of neutral particles, thermal conductivity, finite electron inertia, rotation,

viscosity, magnetic field, permeability, porosity of the medium and radiative heat-loss functions. It is noted that the equation (44) is independent of Hall parameter in other words we can say that there is no influence of Hall current in transverse direction of propagation. In the absence of viscosity, permeability, porosity of the medium, rotation, and partially-ionized plasmas, (44) reduces to that of Bora and Talwar [16] in dimensional form.

3.7 Axis of Rotation Along Magnetic Field

When the axis of rotation is along the magnetic field, we put $\Omega_x = 0$, and $\Omega_z = \Omega$ in the dispersion relation (44) and this gives.

$$M^2 \left(\sigma M^2 + \sigma M \frac{V^2 k^2}{a} + M \Omega_T^2 + 4\sigma \Omega^2 \right) = 0. \quad (45)$$

This dispersion relation (45) shows the simultaneous influence of viscosity, permeability, rotation, thermal conductivity, radiative heat-loss functions and porosity of the medium on the self-gravitational instability of the hydromagnetic fluid plasma. Dispersion relation (45) has two different independent modes of propagation corresponding to equations.

$$\sigma^2 + \sigma P + \Omega_v \nu_c = 0. \quad (46)$$

$$\sigma^7 + \beta_1 \sigma^6 + \sigma^5 \beta_2 + \sigma^4 \beta_3 + \sigma^3 \beta_4 + \sigma^2 \beta_5 + \sigma \beta_6 + \beta_7 = 0. \quad (47)$$

where

$$\beta_1 = \left[\frac{\Omega_m}{\alpha} + 2P + B \right].$$

$$\beta_2 = \left[\frac{\Omega_m}{\alpha} (2P + B) + 2\Omega_v \nu_c + P(P + 2B) + \frac{V^2 k^2}{\alpha} + \frac{\Omega_j^2}{\varepsilon} + 4\Omega^2 \right].$$

$$\beta_3 = \left[\frac{\Omega_m}{\alpha} \left(\frac{\Omega_j^2}{\varepsilon} + P^2 + 2EB + 4\Omega^2 \right) + 2\Omega_v \nu_c \left(P + B + \frac{\Omega_m}{\alpha} \right) + P \left(\frac{\Omega_j^2}{\varepsilon} + PB \right) + k^2 \nu 2\alpha P + B + \nu c + \Omega / 2\varepsilon + \nu c \Omega_j / 2\varepsilon + 4\Omega 2B + 8\Omega 2\nu c. \right]$$

$$\beta_4 = \left[\frac{\Omega_m}{v} \left(\frac{\Omega_j^2}{\varepsilon} + P^2 B + P + \frac{\nu c \Omega_j^2}{\varepsilon} + 4\Omega^2 B \right) + \frac{V^2 k^2}{\alpha} (PB + B\nu c + P\nu c) + \nu c \Omega_v \Omega_v \nu c + 2PB + 2P\Omega_m \alpha + 2B\Omega_m \alpha + k^2 \nu 2\alpha + \Omega_j / 2\varepsilon + P\Omega / 2\varepsilon + \nu c P\Omega_j / 2\varepsilon + \Omega / 2\varepsilon + 4\Omega 2\nu c + 8\Omega 2B + 8\Omega 2\Omega_m \alpha. \right]$$

$$\beta_5 = \left[\frac{\Omega_m}{\alpha} \left(\frac{\nu c \Omega_j^2}{\varepsilon} + \frac{P\nu c \Omega_j^2}{\varepsilon} + \frac{P\Omega_j^2}{\varepsilon} \right) + \Omega_v \nu_c \left(\frac{\Omega_v \nu_c \Omega_m}{\alpha} + \Omega_v \nu_c B + \frac{2PB\Omega_m}{\alpha} + \frac{BV^2 k^2}{\alpha} + \frac{\nu c V^2 k^2}{\alpha} + \Omega_l^2 + \frac{\Omega_m \Omega_j^2}{\varepsilon \alpha} \right) + \nu_c \left(\frac{\Omega_v \nu_c \Omega_j^2}{\varepsilon} + \frac{PB V^2 k^2}{\alpha} + \frac{P\Omega_j^2}{\varepsilon} + \frac{4\Omega^2 \nu c \Omega_m}{\alpha} + 4\Omega^2 B \nu c + \frac{8\Omega^2 B \Omega_m}{\alpha} \right) \right]$$

$$\beta_6 = \left[\Omega_v \nu_c \frac{\Omega_m}{\alpha} \left(\frac{\Omega_m}{\alpha} B \Omega_v \nu_c + \nu_c \frac{BV^2 k^2}{\alpha} + \frac{\Omega_m \Omega_I^2}{\varepsilon \alpha} + \frac{\nu_c \Omega_I^2}{\varepsilon} + \frac{\Omega_m \nu_c \Omega_J^2}{\varepsilon \alpha} + \frac{4 \Omega_m \Omega^2 B \nu_c}{\alpha} \right) + P \nu_c \Omega_m \Omega_I 2 \alpha \right]$$

$$\beta_7 = \left(\frac{\nu_c^2 \Omega_v \Omega_m \Omega_I^2}{\varepsilon \alpha} \right).$$

The first of these, is identical with equation (20) and represents a viscous type of damped stable mode modified by the effects of viscosity collision frequency.

The second one, (47) represents the effect of simultaneous inclusion of the viscosity, thermal conductivity, radiative heat-loss function, permeability, porosity, collision frequency, and rotation on the magneto-gravitational instability of plasma medium when the wave propagation is assumed to be perpendicular to the prevalent magnetic field. It can be seen that when $\Omega_I^2 < 0$, the constant term of the dispersion relation (47) will be negative. This implies that at least one root of is positive, hence the system is unstable. So the condition of instability for transverse mode of propagation is given as

$$\Omega_I^2 = \left[k^2 \left(T_0 \mathcal{L}_T - \rho_0 \mathcal{L}_\rho + \frac{\lambda k^2 T_0}{\rho_0} \right) - 4 \pi G \rho_0 \left(\frac{T_0 \rho_0 \mathcal{L}_T}{p_0} + \frac{T_0 \lambda k^2}{p_0} \right) \right] < 0. \quad (48)$$

This condition of instability in transverse mode of propagation is identical to the condition of instability (33) for longitudinal mode of propagation, which has already been discussed. In the absence of viscosity and neutral particles

$$\sigma^4 + \sigma^3 \left[\frac{\Omega_m}{\alpha} + B \right] + \sigma^2 \left[\frac{\Omega_m}{\alpha} B + \frac{\nu^2 k^2}{\alpha} + \frac{\Omega_J^2}{\varepsilon} + 4 \Omega^2 \right] + \sigma \left[\frac{\Omega_m}{\alpha} \left(\frac{\Omega_J^2}{\varepsilon} + 4 \Omega^2 \right) + k 2 \nu 2 \alpha B + \Omega_I 2 \varepsilon + 4 \Omega 2 B + \Omega m \alpha \Omega_I 2 \varepsilon + 4 \Omega 2 B \right] = 0. \quad (49)$$

The instability of the system in this case will be governed by the condition $(\Omega_I^2 + 4 \Omega^2 B \varepsilon) < 0$; i.e., the system will be unstable for all $k < k_{j11}$, where

$$\sqrt{2} k_{j11} = \sqrt{X_3 \pm [X_3^2 + Y_3]^{1/2}}, \quad (50)$$

$$X_3 = \left(\frac{\rho_0^2 \mathcal{L}_\rho}{\lambda T_0} + \frac{4 \pi G \rho_0}{S_t^2} - \frac{\rho_0 \mathcal{L}_T}{\lambda} - \frac{4 \varepsilon \Omega^2}{S_t^2} \right), \quad Y_3 = \frac{16 \mathcal{L}_T}{\lambda S_t^2} (\pi G \rho_0^2 - \varepsilon \Omega^2).$$

This is the modified condition of instability of radiative instability due to the effect of rotation. From (50) we conclude that rotation decreases the value of critical wave number and tries to stabilize the. Here we notice that the rotation affects the radiative instability criterion in transverse mode of propagation when medium is in-viscid and axis of rotation is taken parallel to the magnetic field. It means that the viscosity parameter removes the effect of rotation. Again in the absence of electrical resistivity we can write (47) in the form.

$$\sigma^3 + \sigma^2 B + \sigma \left[\frac{V^2 k^2}{\alpha} + \frac{\Omega_f^2}{\varepsilon} + 4\Omega^2 \right] + \left[\frac{k^2 v^2 B}{\alpha} + \frac{\Omega_f^2}{\varepsilon} + 4\Omega^2 B \right] = 0. \quad (51)$$

Here we get the condition of instability if $\left(\Omega_l^2 + 4\Omega^2 B \varepsilon + \frac{k^2 v^2 B \varepsilon}{\alpha} \right) < 0$. The system will be unstable for all $k < k_{j11}$ where

$$k_{j12} = \sqrt{\frac{X_4 \pm [X_4^2 + Y_4]^{1/2}}{2}}, \quad (52)$$

$$X_4 = \left[\left(\frac{\rho_0^2 \mathcal{L}_\rho}{\lambda T_0} + \frac{4\pi G \rho_0}{S_l^2} - \frac{4\varepsilon \Omega^2}{S_l^2} \right) \left(1 + \frac{\varepsilon V^2}{\chi S_l^2} \right)^{-1} - \frac{\rho_0 \mathcal{L}_T}{\lambda} \right], \quad Y_4 = \frac{16\mathcal{L}_T}{\lambda S_l^2} (\pi G \rho_0^2 - \varepsilon \Omega^2).$$

Reviewing the condition of instability (52), i.e. the value of the critical wave number, we can conclude that presence of magnetic field and rotation modifies the condition of instability. Hall current does not affect the condition of instability in this mode of propagation. It is also noted that when the medium is finitely electrical conducting the effect of magnetic field, in condition of instability, vanishes.

For our convenience to show a better insight, the graphical presentation of the exact growth rate of the system represented by (47) can be written in non-dimensional form (Appendix) introducing dimensionless quantities, assuming $(\rho \gg \rho_d)$ so that $b \ll 1$ and dividing (47) by $(4\pi G \rho_0)^{1/2}$, as

$$\begin{aligned} \sigma^* &= \frac{\sigma}{(4\pi G \rho_0)^{1/2}}, \quad v_c^* = \frac{v_c}{(4\pi G \rho_0)^{1/2}}, \quad k^* = \frac{k s_a}{(4\pi G \rho_0)^{1/2}}, \quad v^* = \frac{v(4\pi G \rho)^{1/2}}{s_a^2}, \\ \lambda^* &= \frac{(\gamma-1)T\lambda(4\pi G \rho_0)^{1/2}}{p s_a^2}, \quad \eta^* = \frac{\eta(4\pi G \rho_0)^{1/2}}{s_a^2}, \quad \Omega^* = \frac{\Omega}{(4\pi G \rho_0)^{1/2}}, \quad V^* = \frac{V(4\pi G \rho_0)^{1/2}}{s_a}, \\ \mathcal{L}_\rho^* &= \frac{(\gamma-1)\rho \mathcal{L}_\rho}{s_a^2 (4\pi G \rho_0)^{1/2}}, \quad \mathcal{L}_T^* = \frac{(\gamma-1)\rho^T \mathcal{L}_T}{\rho (4\pi G \rho_0)^{1/2}}, \quad \Omega_v^* = k^{*2} v^*, \quad A^* = \frac{1}{\gamma} (\mathcal{L}_T^* + \lambda^* k^{*2}) - \mathcal{L}_\rho^*, \\ B^* &= (\mathcal{L}_T^* + \lambda^* k^{*2}), \quad \Omega_l^{*2} = k^{*2} A^* - B^*, \quad \Omega_j^{*2} = k^{*2} - 1, \quad \Omega_m^* = k^{*2} \eta^* \end{aligned}$$

Numerical calculations were performed to determine the roots of σ from dispersion relation (47), as a function of wave number k for several values of different parameters involved, taking $\gamma = 5/3$. The variations in the growth rate σ^* , with wave number k^* are shown in Figs 1-7.

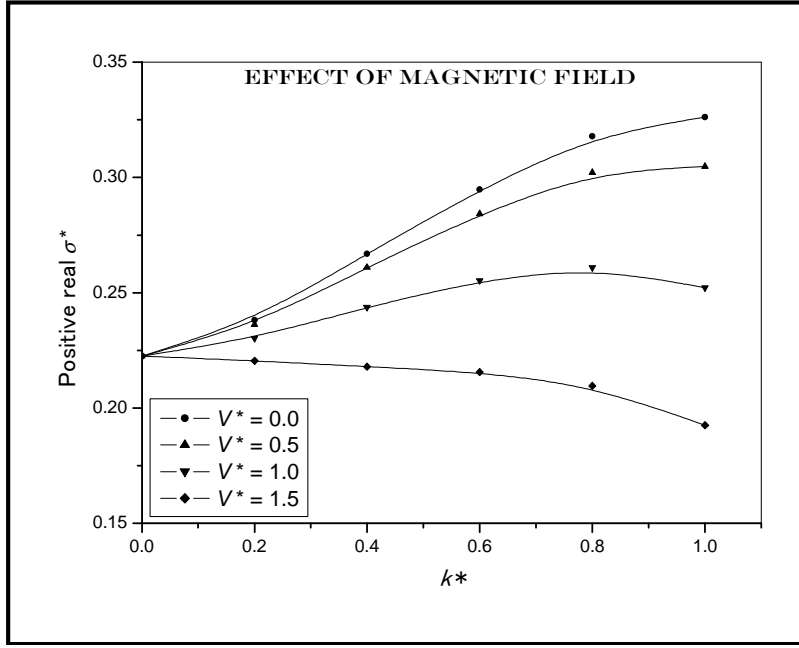


Fig. 1. The growth rate is plotted against the non-dimensional wave number k^* with variation in the normalized magnetic field $V^* = 0.0, 0.5, 1.0, 1.5$ the value $\lambda^* = \nu^* = \nu_c^* = \eta^* = \Omega^* = 1$ and the value of $\mathcal{L}_T^* = 0.0$ and $\mathcal{L}_\rho^* = 0.5$

Fig. 1 shows the variation in growth rate with respect to magnetic field. Here we notice that when the system is unmagnetized the growth of instability is maximum while the growth rate decreases with the increasing value of magnetic field. Thus from the graph we conclude that the effect of magnetic field is to stabilize the system.

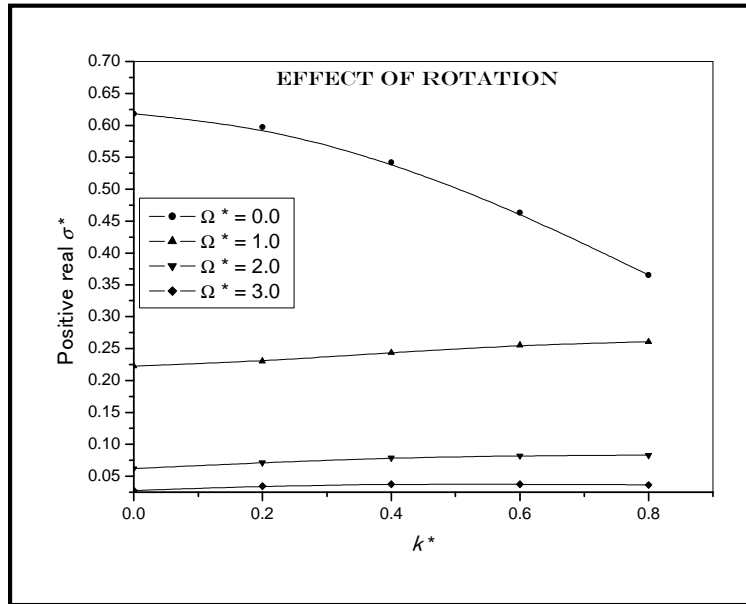


Fig. 2. The growth rate is plotted against the non-dimensional wave number k^* with variation in the normalized rotational effect $\Omega^* = 0.0, 1.0, 2.0, 3.0$ the value $\lambda^* = \nu^* = \nu_c^* = \eta^* = V^* = 1$ and the value of $\mathcal{L}_T^* = 0.0$ and $\mathcal{L}_\rho^* = 0.5$.

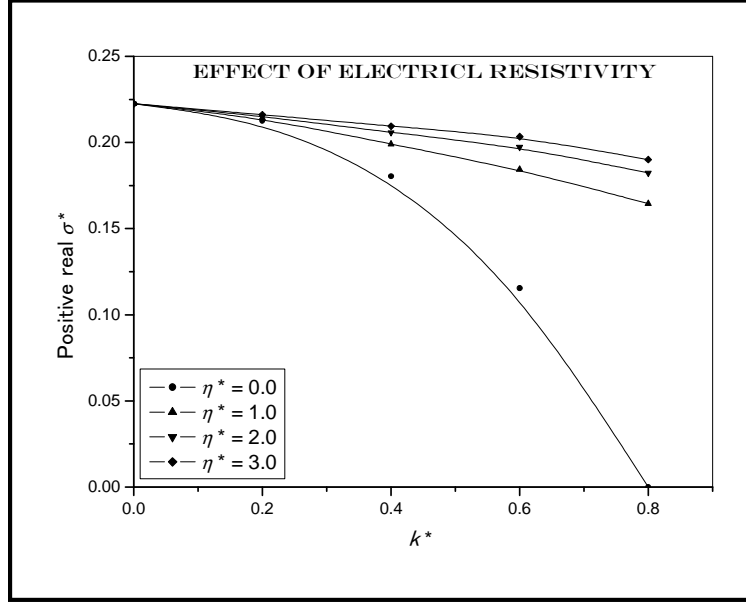


Fig. 3. The growth rate is plotted against the non-dimensional wave number k^* with variation in the normalized resistivity effects $\eta^* = 0.0, 1.0, 2.0, 3.0$ the value $\lambda^* = \nu^* = \nu_c^* = V^* = \Omega^* = 1$ and the value of $\mathcal{L}_T^* = 0.5$ and $\mathcal{L}_\rho^* = 0.0$

Fig. 2 depicts the growth rate of instability in a rotating system against wave vector with variation in rotation. In Fig. 2, the growth rate of instability is maximum for non rotating system and showing decreasing growth rate with increase in value of rotation. It means that rotations decreases the growth rate of instability and try to maintain the stability of the system.

Fig. 3, represent the growth rate v/s wave number with varying values of electrical resistivity. Here on observing the behavior of fig 3 we can say that electrical resistivity increases the growth rate of instability and destabilize the system equilibrium.

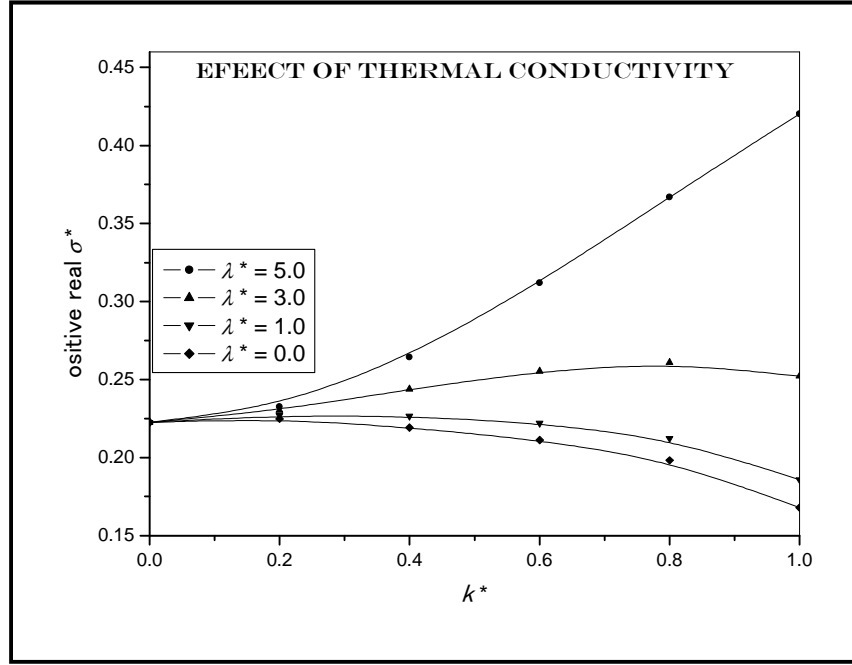


Fig. 4. The growth rate is plotted against the non-dimensional wave number k^* with variation in the normalized thermal conductivity effects $\lambda^* = 0.0, 1.0, 3.0, 5.0$ the value $V^* = \nu^* = \nu_c^* = \eta^* = \Omega^* = 1$ and the value of $\mathcal{L}_T^* = 0.0$ and $\mathcal{L}_p^* = 0.5$.

Fig. 4 shows the effect of thermal conductivity on the growth rate of instability. Here we see that the increasing value of thermal conductivity increases the growth rate of instability. Thus, the thermal conductivity shows a destabilizing effect, reciprocal to the effect of magnetic field and rotation on the growth rate of instability and destabilizes the system.

Fig. 5 is plotted between growth rate and wave number with varying values of collision frequency. From Fig. 5, we can analyze that increasing values of collision frequency decreases the growth rate of the system. In other words we can say that the presence of neutral particles in ionized plasma is to stabilize the equilibrium of system.

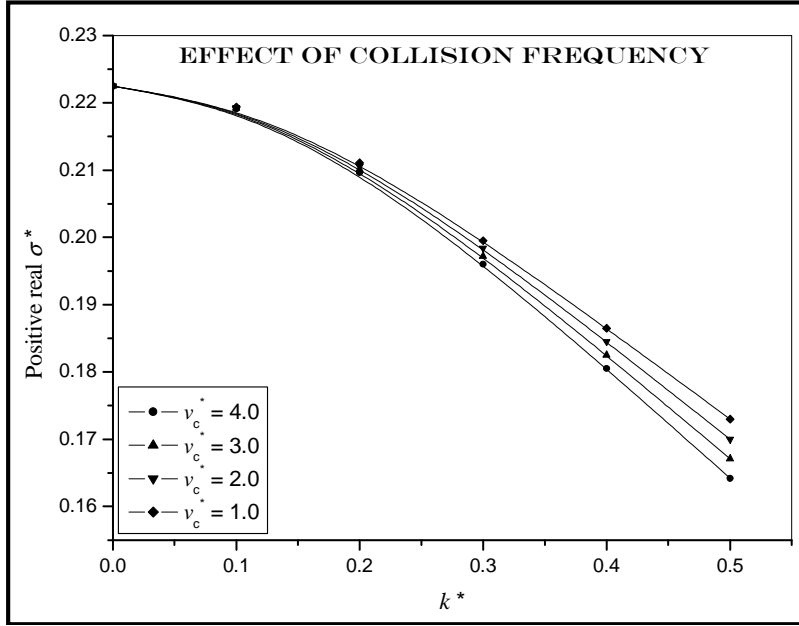


Fig. 5. The growth rate is plotted against the non-dimensional wave number k^* with variation in the normalized neutral particle effects $\nu_c^* = 0.1, 0.2, 0.3, 0.4$ the value $V^* = \nu^* = \lambda^* = \eta^* = \Omega^* = 1$ and the value of $\mathcal{L}_T^* = 0.5$ and $\mathcal{L}_\rho^* = 0.0$.

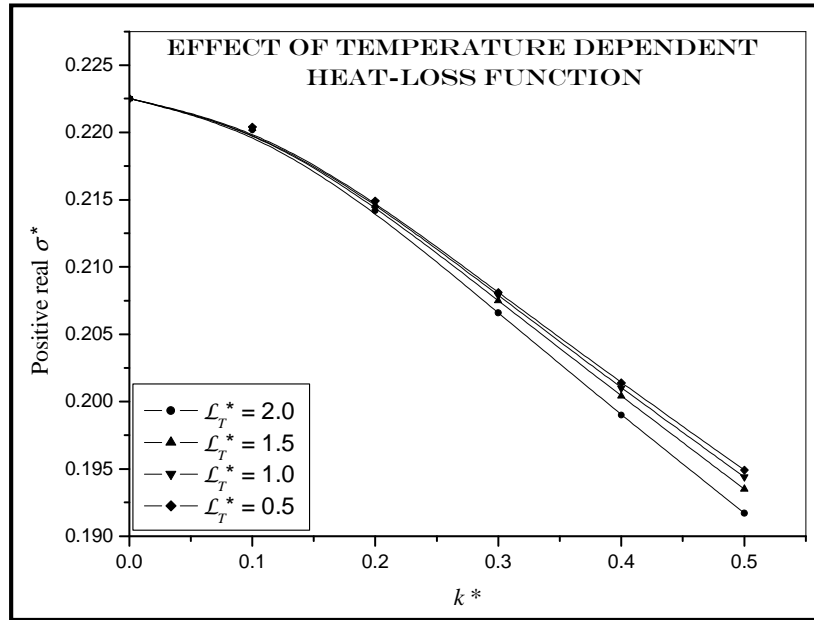


Fig. 6. The growth rate is plotted against the non-dimensional wave number k^* with variation in the normalized temperature dependent hit loss function effects the value $\mathcal{L}_T^* = 0.5, 1.0, 1.5, 2.0$ the value $V^* = \nu_c^* = \nu^* = \lambda^* = \eta^* = \Omega^* = 1$ and the value of $\mathcal{L}_\rho^* = 0.0$. Fig. 6 shows the variation in growth rate with respect to wave number under effect of density dependent heat-loss function. From Fig. (6) we found that the increasing values of

temperature dependent heat-loss function increases the growth rate of instability of considered system. It means that the temperature dependent heat-loss function has a stabilizing effect on the system.

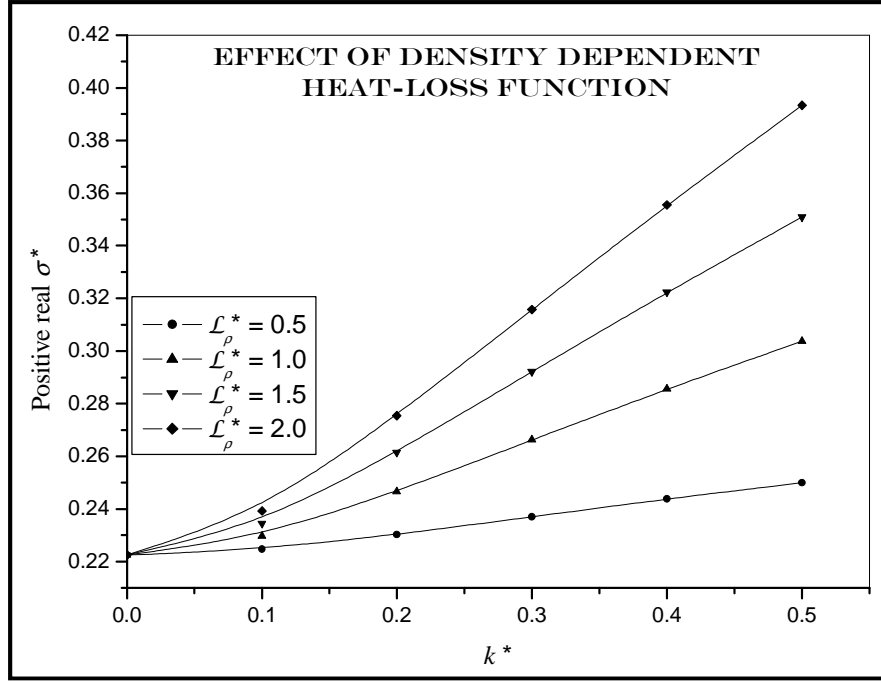


Fig. 7. The growth rate is plotted against the non-dimensional wave number k^* with variation in the normalized temperature dependent hit loss function effects the value $\mathcal{L}_\rho^* = 0.5, 1.0, 1.5, 2.0$ the value $V^* = \nu_c^* = \nu^* = \lambda^* = \eta^* = \Omega^* = 1$ and the value of $\mathcal{L}_T^* = 0.0$.

Fig. 7 is plotted, to show the effect of density dependent heat-loss function, between growth rate and wave number. From figure (7) we analyze that the density dependent heat-loss function plays a same role as thermal conductivity and electrical resistivity play to destabilize the system. It means that the increasing values of density dependent heat-loss function increases the growth rate of instability.

3.8 Non-Gravitating Hydromagnetic Fluid

In the transverse mode of propagation, dispersion relation (45), for non-gravitating hydromagnetic fluid; i.e. $(4\pi G\rho_0 = 0)$ has two independent factors. First factor is identical to (20) and shows a viscous damped mode while the last factor (47) is the seventh degree polynomial equation from which the constant term of last coefficients gives the condition of instability as

$$A = \left(\mathcal{L}_T T_0 - \mathcal{L}_\rho \rho_0 + \frac{\lambda k^2 T_0}{\rho_0} \right) < 0. \quad (53)$$

This condition of instability is similar to the condition of thermal instability obtained by Field [15] and also to the conditions of instability (37) in longitudinal mode of propagation for non-

gravitating hydromagnetic fluid. Now for perfectly conducting and in viscid fluid this condition of instability will be modify as

$$k^2 \left(\mathcal{L}_T T_0 - \mathcal{L}_\rho \rho_0 + \frac{\lambda k^2 T_0}{\rho_0} \right) + \left\{ \frac{k^2 V^2}{\alpha} + 4\Omega^2 \right\} \varepsilon \left(\frac{\mathcal{L}_T T_0 \rho_0}{p_0} + \frac{\lambda k^2 T_0}{p_0} \right) < 0. \quad (54)$$

From the above condition of instability we can say that magnetic field and rotation modifies the condition of instability in transverse mode of propagation for infinitely electrical conducting and in viscid fluid when axis of rotation is along the magnetic field. It means that in our case we find the modified condition of thermal instability due to presence of rotation and magnetic field.

3.9 Axis of Rotation Perpendicular To The Magnetic Field

We now analyze the wave propagation in transverse direction of external magnetic field considering the rotation of the magnetic field, we put $\Omega_x = \Omega$ and $\Omega_z = 0$, the dispersion relation (44) reduces to

$$-M(M^2 + 4\Omega^2) \left[\sigma M + \sigma \frac{V^2 k^2}{a} + \Omega_T^2 \right] = 0. \quad (55)$$

Equation (55) has three independent factors, each representing a different mode of propagation. The first of these, identical with equation (20) and represents a viscous type of damped stable mode modified by the effects of viscosity collision frequency. The second factor of equation (55) equating to zero, gives

$$\sigma^4 + \sigma^3 2P + \sigma^2 [2\nu_c \Omega_v + P^2 + 4\Omega^2] + \sigma \nu_c [2P\Omega_v + 8\Omega^2] + \nu_c^2 (\Omega_v^2 + 4\Omega^2) = 0. \quad (56)$$

This dispersion relation shows a rotating mode with the effect of collision frequency, viscosity and permeability of the porous plasma medium, which is independent of thermal conductivity, finite electrical conductivity, finite electron inertia, Hall current, and radiative heat-loss function. Equation (56) is a forth degree polynomial having all the coefficients positive and a positive absolute term. So the equation will have all the four roots either negative or complex conjugates with negative real; i.e., it will represent a stable mode. In the absence of neutral particles and viscosity we get

$$\sigma^2 + 4\Omega^2 = 0. \quad (57)$$

This represents a purely rotational mode which is oscillatory and stable in nature. Hence it is obvious that rotation in this direction of propagation does not alter the condition of instability but gives a separate stable mode. The presence of neutral particles, permeability and viscosity simply modifies this mode. The third factor of equation (55) equating to zero gives.

$$\sigma^5 + A_1 \sigma^4 + \sigma^3 A_2 + \sigma^2 A_3 + \sigma A_4 + A_5 = 0. \quad (58)$$

where,

$$A_1 = \left[\frac{\Omega_m}{\alpha} + P + B \right].$$

$$\begin{aligned}
A_2 &= \left[\frac{\Omega_m}{\alpha} (P + B) + PB + \Omega_v \nu_c + \frac{V^2 k^2}{\alpha} + \frac{\Omega_I^2}{\varepsilon} \right]. \\
A_3 &= \left[\frac{\Omega_m}{\alpha} \left(\frac{\Omega_I^2}{\varepsilon} + PB \right) + \Omega_v \nu_c \left(B + \frac{\Omega_m}{\alpha} \right) + \frac{k^2 V^2}{\alpha} (B + \nu_c) + \frac{\Omega_I^2}{\varepsilon} + \frac{\nu_c \Omega_I^2}{\varepsilon} \right]. \\
A_4 &= \left[\frac{\Omega_m}{\alpha} \left(\Omega_v \nu_c B + \frac{\Omega_v \Omega_I^2}{\varepsilon} + \Omega_I^2 \right) + \frac{V^2 k^2 B \nu_c}{\alpha} + \frac{\nu_c \Omega_I^2}{\varepsilon} \right]. \\
A_5 &= \left(\frac{\nu_c \Omega_m \Omega_I^2}{\varepsilon \alpha} \right).
\end{aligned}$$

Equation (58) represents the dispersion relation for transverse wave propagating through an infinite homogeneous, self-gravitating, viscous magnetized partially ionized plasma having finite electrical resistivity, rotation, radiative effects, with the effect of neutral particles. it can be seen that when $\Omega_I^2 < 0$, The constant term A_5 of the dispersion relation (58) will be negative. This implies that at least one root of (55) is positive, hence the system is unstable. So the condition of instability for such case in transverse mode of propagation is given as

$$\Omega_I^2 = k^2 A - 4\pi G \rho_0 B < 0. \quad (59)$$

Which is the same condition of instability discussed in (33) and obtained by Bora and Talwar [16] for finitely electrical conducting, self-gravitating plasma in transverse mode of propagation. Now in the absence of collision frequency between two components of plasma, kinematic viscosity, permeability and electrical resistivity i.e. $\eta = 0, \nu = 0, \nu_c = 0$, and $K_1 = 0$ the dispersion relation (58) reduces to as

$$\sigma^3 + B\sigma^2 + \sigma \left(\frac{V^2 k^2}{\alpha} + \frac{\Omega_I^2}{\varepsilon} \right) + \frac{BV^2 k^2}{\alpha} + \frac{\Omega_I^2}{\varepsilon} = 0. \quad (60)$$

Equation (60) represents a dispersion relation for infinite homogeneous, self-gravitating, thermally conducting plasma with radiative heat-loss effects. The condition of instability for such case is obtained from the constant term of equation (60), is given as

$$\left(\frac{BV^2 k^2 \varepsilon}{\alpha} + k^2 A - 4\pi G \rho_0 B \right) < 0. \quad (61)$$

This is modified condition of radiative instability due to the effect of magnetic field, electron inertia, porosity of the medium, thermal conductivity and radiative heat-loss function in transverse mode of propagation. The condition is identical with condition obtained by Patidar *et al.* [25] and also by Aggrawal and Talwar [27]. From these conditions it is clear that if the fluid expressed by equation (61) does not contain radiative heat-loss function then the critical Jeans wave number below which the system is unstable is obtained from the constant terms of equation (61) and is given as

$$k_{j5}^2 = \frac{\gamma k_j^2}{\sqrt{\left(1 + \gamma \frac{\varepsilon V^2}{f c^2}\right)}}. \quad (62)$$

If the arbitrary radiative heat-loss functions are included in a thermally non conducting medium, the corresponding value of critical wave number is given by

$$k_{j6}^2 = \gamma k_j^2 \left[\frac{TL_T}{TL_T(1+\gamma\frac{\varepsilon v^2}{fc^2}) - L_\rho \rho} \right]^{1/2}. \quad (63)$$

The disturbances with a wave number $k < k_{j6}$ are unstable, where for $k > k_{j6}$, the disturbances are stable. If the fluid expressed by equation (60) is assumed to be unmagnetized i.e. $H = 0$ then the dispersion relation becomes for such case as

$$\sigma^3 + B\sigma^2 + \sigma \frac{\Omega_I^2}{\varepsilon} + \frac{\Omega_I^2}{\varepsilon} = 0. \quad (64)$$

This is the dispersion relation for infinite homogeneous non-magnetized, self-gravitating, thermally conducting plasma having electron inertia, porosity and radiative effect. Condition of instability for this case is given as

$$\Omega_I^2 = k^2 A - 4\pi G \rho_0 B < 0. \quad (65)$$

This condition is identical to (48). On comparing equation (61) and (65) we find that contribution of electron inertia and porosity in the condition of instability is effective only when the considered fluid is magnetized. The effect of magnetic field comes through the term $V^2 k^2 B$ of magnetic field, there is an upward shift in the instability threshold i.e. the magnetic field decreases the value of critical wave number. Thus, we conclude that the magnetic field stabilizes the medium for transverse propagation.

3.10 Non-Gravitating Hydromagnetic Fluid

For this condition, first two modes of propagation is similar the two modes (20) and (56) of the dispersion relation (55) for transverse propagation, when axis of rotation is perpendicular to a magnetic field but the third factor, for perfectly electrical conducting medium, can be written as

$$\sigma^4 + \sigma^3 [P + B] + \sigma^2 \left[PB + \Omega_v \nu_c + \frac{V^2 k^2}{\alpha} + \frac{S_a^2 k^2}{\varepsilon} \right] + \sigma \left[\Omega_v \nu_c B + \frac{k^2 V^2}{\alpha} (B + \nu_c) + \frac{k^2 A \varepsilon + \nu_c S_a 2k^2 \varepsilon + \nu_c V^2 2k^2 B \alpha + k^2 A \varepsilon}{\varepsilon} \right] = 0. \quad (66)$$

Equation (66) represents the combined influence of thermal conductivity, radiative heat-loss function, and magnetic field on the instability of two components partially-ionized plasmas with the effect of viscosity and permeability of the porous medium. The condition of instability is obtained from dispersion relation (66) as

$$BV^2 \varepsilon + \alpha A < 0. \quad (67)$$

This is modified condition of thermal instability due to magnetic field, finite electron inertia, and porosity of the medium. From equation (27) the expression for critical wave number will be given as

$$k_{j7}^2 = \left[\left\{ \frac{\rho^2 \mathcal{L}_\rho}{\lambda T} - \frac{\rho \mathcal{L}_T}{\lambda} \left(1 + \frac{\varepsilon v^2}{\alpha c^2} \right) \right\} / \left(1 + \frac{\varepsilon v^2}{\alpha c^2} \right) \right]. \quad (68)$$

The medium is unstable for wave number $k < k_{j7}$. It may be noted here that the critical wave number involves, derivative of temperature dependent and density dependent arbitrary radiative heat-loss function, thermal conductivity of the medium and the magnetic field.

4. CONCLUSIONS

The magneto-thermal instability of a rotating self-gravitating partially ionized Hall plasma permeated by a magnetic field has been investigated in the presence of the effects of electrical resistivity, finite electron inertia, porosity, permeability and viscosity of the medium. The general dispersion relation is obtained using normal mode analysis. This general dispersion relation is discussed for longitudinal and transverse modes of propagation for each cases when axis of rotation taking along and perpendicular to the magnetic field. In general, we find that the Jeans condition remains valid but the expression of the critical Jeans wave number is modified due to presence of thermal conductivity and radiative heat-loss function. Numerical calculations have been performed, in transverse mode of propagation, to obtain the dependence of the growth rate of the gravitational unstable mode on the various physical effects. We found that

- (1) Viscosity, permeability of porous medium and collision frequency of the two component partially ionized plasma have stabilizing effects in both longitudinal and transverse mode of propagation. Also it is found that the direction of axis of rotation, do not affect the stabilizing effects of these parameter.
- (2) Angular frequency of rotation has stabilizing effects in transverse mode of propagation when axis of rotation is along the magnetic field and fluid is in-viscid.
- (3) The electrical resistivity has a destabilizing effect. Also in the transverse mode of propagation electrical resistivity eliminate the effect of magnetic field from the condition of radiative instability and increase the value of critical wave number.
- (4) The magnetic field modifies the radiative instability criterion in the transverse mode of propagation.
- (5) The Hall current parameter does not affect the condition of instability but has a destabilizing effect in longitudinal mode of propagation.
- (6) Finite electron inertia modifies the growth rate of the instability in longitudinal as well as transverse mode of propagation. Also finite electron inertia, in transverse mode of propagation, modifies the condition of radiative instability when external magnetic field is present.
- (7) Porosity of the medium stabilizes the system by reducing the critical wave number in a rotating or a magnetized or a rotating magnetized medium.

From the nature of the growth rate of instability presented in Figs. 1-7 with variation in various parameters we can conclude that the thermal conductivity, electrical resistivity and density-dependent heat-loss function have destabilizing influence on the instability of the fluid. It is also observed that the contribution of rotation, magnetic field, viscosity and collision frequency it to reduce the growth rate and stabilize the system.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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Appendix A

Non-dimensional form of (47)

$$\sigma^{*7} + \sigma^{*6}(\Omega_m^* + 2\Omega_v^* + 4v_c^* + \beta^*) + \sigma^{*5}[\Omega_m^*(2\Omega_v^* + 4v_c^* + \beta^*) + 2v_c^*(3\Omega_v^* + 2v_c^* + 2\beta^*) + v^{*2}k^{*2} + \Omega_j^{*2} + 4\Omega^{*2} + 2\beta^*\Omega_v^* + \Omega_v^{*2}] + \sigma^{*4}[\Omega_m^*(\Omega_j^{*2} + 2\beta^*\Omega_v^* + 4\beta^*v_c^* + 4\Omega^{*2} + 4v_c^{*2} + \Omega_v^{*2} +$$

$$\begin{aligned}
& 4\Omega_v^* v_c^*) + 2\Omega_v^* v_c^* (\Omega_m^* + \Omega_v^* + 2v_c^* + \beta^*) + V^{*2} k^{*2} (\Omega_v^* + 3v_c^* + \beta^*) + v_c^* (3\Omega_j^{*2} + 4\beta^* \Omega_v^* + \\
& 4\beta^* v_c^* + 8\Omega^{*2}) + \Omega_j^{*2} + 4\Omega^{*2} + \Omega_v^* (\Omega_j^{*2} + \Omega_v^* \beta^*) + \sigma^{*3} [\Omega_m^* (\Omega_l^{*2} + 4v_c^{*2} \beta^* + \Omega_v^* \beta^* + 4\beta^* v_c^* \Omega_v^* + \\
& 2v_c^* \Omega_j^{*2} + \Omega_v^* \Omega_j^{*2} + v_c^* \Omega_j^{*2} + 4\Omega^{*2} \beta^*) + V^{*2} k^{*2} (\beta^* \Omega_v^* + 3\beta^* v_c^* + 2v_c^{*2} + \Omega_v^* v_c^*) + \Omega_v^* v_c^* (v_c^* \Omega_v^* + \\
& 2\beta^* \Omega_v^* + 4v_c^* \Omega_m^* + 4\beta^* v_c^* + 2\Omega_m^* \Omega_v^* + 2\beta^* \Omega_m^* + 2\beta^* \Omega_m^* + k^{*2} V^{*2} + \Omega_j^{*2}) + v_c^* (2v_c^* \Omega_j^{*2} + \\
& \Omega_v^* \Omega_j^{*2} + \Omega_l^{*2} + 4\Omega^{*2} v_c^* + 8\Omega^{*2} \beta^* + 8\Omega^{*2} \Omega_m^*) + 2v_c^* \Omega_l^{*2} + \Omega_v^* \Omega_l^{*2}] + \sigma^{*2} [\Omega_m^* (2v_c^* \Omega_j^{*2} + \\
& \Omega_v^* v_c^* \Omega_j^{*2} + 3v_c^* \Omega_l^{*2} + \Omega_v^* \Omega_l^{*2}) + \Omega_v^* v_c^* (v_c^* \Omega_m^* \Omega_v^* + \Omega_v^* v_c^* \beta^* + 4v_c^* \beta^* \Omega_m^* + 2\Omega_v^* \beta^* \Omega_m^* + \beta^* V^{*2} k^{*2} + \\
& v_c^* V^{*2} k^{*2} + \Omega_l^{*2} + \Omega_m^* \Omega_j^{*2}) + v_c^* (\Omega_v^* v_c^* \Omega_j^{*2} + 2v_c^* \beta^* V^{*2} k^{*2} + \Omega_v^* \beta^* V^{*2} k^{*2} + 2v_c^* \Omega_l^{*2} + \Omega_v^* \Omega_l^{*2} + \\
& 4\Omega^{*2} v_c^* \Omega_m^* + 4\Omega^{*2} \beta^* v_c^* + 8\Omega^{*2} \beta^* \Omega_m^*)] + \sigma^* [\Omega_v^* v_c^* \Omega_m^* (\Omega_m^* \beta^* \Omega_v^* v_c^* + v_c^* V^{*2} k^{*2} \beta^* + \Omega_m^* \Omega_l^{*2} + \\
& v_c^* \Omega_l^{*2} + \Omega_m^* v_c^* \Omega_j^{*2} + 4\Omega_m^* \Omega^{*2} \beta^* v_c^*) + 2v_c^* \Omega_l^{*2} \Omega_m^* + \Omega_v^* v_c^* \Omega_m^* \Omega_l^{*2}] + (v_c^* \Omega_v^* \Omega_m^* \Omega_l^{*2}).
\end{aligned}$$

Appendix B

$$\alpha_1 = \left(4 \frac{\Omega_m}{\alpha} + 2P \right).$$

$$\alpha_2 = \left\{ \frac{\Omega_m}{\alpha} \left(6 \frac{\Omega_m}{\alpha} + 8P \right) + \frac{2Q^2 k^4}{\alpha^2} + 2\Omega_v v_c + P^2 + \frac{2V^2 k^2}{\alpha} + 4\Omega^2 \right\}.$$

$$\alpha_3 = \left\{ \frac{\Omega_m}{\alpha} \left(12F \frac{\Omega_m}{\alpha} + 4 \frac{\Omega_m^2}{\alpha^2} + 4 \frac{Q^2 k^4}{\alpha^2} + 4P^2 + 6 \frac{V^2 k^2}{\alpha} + 16\Omega^2 + 8\Omega_v v_c \right) + v_c \left(2\Omega_v P + 8\Omega^2 + 2V^2 k^2 \alpha + 4PQ^2 k^4 \alpha + 2PV^2 k^2 \alpha \right) \right\}.$$

$$\alpha_4 = \left\{ \frac{\Omega_m}{\alpha} \left(8P \frac{\Omega_m^2}{\alpha^2} + \frac{\Omega_m^3}{\alpha^3} + 2 \frac{\Omega_m Q^2 k^4}{\alpha^3} + 8P \frac{Q^2 k^4}{\alpha^2} + 6 \frac{\Omega_m}{\alpha} P^2 + 6\Omega_m \frac{V^2 k^2}{\alpha} + 6P \frac{V^2 k^2}{\alpha} + 24 \frac{\Omega_m}{\alpha} \Omega^2 \right) + \Omega m a v c 12 \Omega m a \Omega v + 8 \Omega v P + 32 \Omega^2 + 6 V^2 k^2 \alpha + v c 2 P V^2 k^2 \alpha + 4 Q^2 k^4 \alpha 2 \Omega v + \Omega v 2 v c + 2 \Omega v V^2 k^2 \alpha + 4 v c \Omega^2 + Q^2 k^4 \alpha 2 Q^2 k^4 \alpha + 2 P^2 + 2 V^2 k^2 \alpha + 8 \Omega^2 + V^4 k^4 \alpha + 4 \Omega V^2 k^4 Q \alpha \right\}.$$

$$\alpha_5 = \left\{ \frac{\Omega_m}{\alpha} \left(2P \frac{\Omega_m^3}{\alpha^3} + 4P \Omega_m \frac{Q^2 k^4}{\alpha^3} + 4P^2 \frac{Q^2 k^4}{\alpha^2} + 4 \frac{\Omega_m^2}{\alpha^2} P^2 + 7P \frac{\Omega_m V^2 k^2}{\alpha^2} + 2 \frac{\Omega_m^2 V^2 k^2}{\alpha^3} + 2 \frac{V^2 k^2 Q^2 k^4}{\alpha^3} + 2V^4 k^4 \alpha + 16 \Omega m^2 a^2 \Omega^2 + 16 \Omega^2 Q^2 k^4 \alpha + 8 \Omega Q V^2 k^4 \alpha + \Omega m a v c 8 \Omega m^2 a^2 \Omega v + 6 V^2 k^2 \Omega m a^2 + 8 \Omega v Q^2 k^4 \alpha + 12 P \Omega m a \Omega v + 4 \Omega v 2 v c + 6 P V^2 k^2 \alpha + 6 \Omega v V^2 k^2 \alpha + 16 v c \Omega^2 + 48 \Omega m a \Omega^2 + v c 4 E Q^2 k^4 \alpha 2 \Omega v + 2 V^2 k^2 a \Omega v v c + 2 V^2 k^2 Q^2 k^4 \alpha + 2 V^4 k^4 \alpha + 16 Q^2 k^4 \alpha 2 \Omega^2 + 8 \Omega Q V^2 k^4 \alpha + 2 P Q^2 k^4 \alpha 2 Q^2 k^4 \alpha + V^2 k^2 \alpha \right) \right\}.$$

$$\alpha_6 = \left\{ \frac{\Omega_m v_c}{\alpha} \left(\frac{2\Omega_v \Omega_m^3}{\alpha^3} + \frac{4\Omega_v Q^2 k^4 \Omega_m}{\alpha^3} + \frac{2Q^2 k^4 V^2 k^2}{\alpha^3} + \frac{8E \Omega_v v_c \Omega_m}{\alpha} + \frac{6P \Omega_m V^2 k^2}{\alpha^2} + 8E \frac{Q^2 k^4}{\alpha^2} \Omega_v + \frac{6\Omega_m \Omega_v^2 v_c}{\alpha} + 6V^2 k^2 v c \Omega v \alpha + 2 \Omega m^2 V^2 k^2 \alpha + 5 \Omega v \Omega m V^2 k^2 \alpha + 4 V^4 k^4 \alpha + 32 \Omega m^2 \Omega^2 \alpha + 24 \Omega m v c \Omega^2 \alpha + 32 Q^2 k^4 \alpha 2 \Omega^2 + 16 \Omega Q V^2 k^4 \alpha + \Omega m a P \Omega m^3 \alpha + 2 P^2 Q^2 k^4 \Omega m \alpha + 2 P \Omega m^2 V^2 k^2 \alpha + 2 P Q^2 k^4 V^2 k^2 \alpha + 2 \Omega m V^4 k^4 \alpha + 4 \Omega m^2 \Omega^2 \alpha + 8 \Omega^2 Q^2 k^4 \Omega m \alpha + 4 Q V^2 k^4 \Omega m \Omega \alpha + v c 2 Q^2 k^4 \alpha 2 \Omega v 2 v c + 2 \Omega v Q^2 k^4 V^2 k^2 \alpha + 2 P Q^2 k^4 V^2 k^2 \alpha + V^4 k^4 v c \alpha + 2 \Omega v Q^4 k^8 \alpha + 8 \Omega^2 Q^2 k^4 \alpha 2 v c + 4 \Omega Q V^2 k^4 v c \alpha + Q^2 k^4 \alpha 2 P Q^2 k^4 \alpha + V^4 k^4 \alpha + 4 \Omega Q V^2 k^4 \alpha + 4 \Omega^2 Q^2 k^4 \alpha \right) \right\}.$$

$$\alpha_7 = \left\{ \frac{\Omega_m v_c}{\alpha} \left(\frac{2P \Omega_v \Omega_m^3}{\alpha^3} + \frac{4P \Omega_v Q^2 k^4 \Omega_m}{\alpha^3} + \frac{2P \Omega_m^2 V^2 k^2}{\alpha^3} + \frac{2P Q^2 k^4 V^2 k^2}{\alpha^3} + \frac{4\Omega_m^2 \Omega_v^2 v_c}{\alpha^2} + \frac{6V^2 k^2 \Omega_m v_c \Omega_v}{\alpha^2} + 4Q^2 k^4 \alpha 2 \Omega v 2 v c + 2 \Omega m^2 \Omega v V^2 k^2 \alpha + 2 V^2 k^2 Q^2 k^4 \Omega v \alpha + 3 \Omega m V^4 k^4 \alpha + 2 V^4 k^4 v c \alpha + 16 \Omega m^2 \Omega^2 v c \alpha + 16 Q^2 k^4 \alpha 2 \Omega^2 v c + 8 \Omega m^3 \Omega^2 \alpha + 16 Q^2 k^4 \Omega^2 m \alpha + 8 v c \Omega v 2 Q V^2 k^4 \alpha + 8 \Omega \Omega m Q V^2 k^4 \alpha + v c 2 P Q^4 k^8 \alpha \Omega v + 2 \Omega v v c Q^2 k^4 V^2 k^2 \alpha + 8 \Omega^2 Q^4 k^8 \alpha + 2 Q^2 k^8 V^4 \alpha + 8 \Omega Q V^2 k^4 Q^2 K^4 \alpha \right) \right\}.$$

$$\alpha_8 = \left\{ \frac{\Omega_m v_c}{\alpha} \left(\frac{v_c \Omega_v^2 \Omega_m^3}{\alpha^3} + \frac{2Q^2 k^4 \Omega_m \Omega_v^2 v_c}{\alpha^3} + \frac{\Omega_m^2 V^2 k^2 v_c \Omega_v}{\alpha^3} + \frac{2v_c \Omega_v Q^2 k^4 V^2 k^2}{\alpha^3} + \frac{\Omega_m^2 v_c \Omega_v V^2 k^2}{\alpha^3} + \frac{V^4 k^4 \Omega_m v_c}{\alpha^3} + 4 \Omega m^3 \Omega^2 v c \alpha + 4 \Omega Q^4 k^4 V^2 \Omega m v c \alpha + 8 \Omega m Q^2 k^4 \Omega^2 v c \alpha + v c v c \Omega v^2 Q^4 k^8 \alpha + 4 \Omega^2 Q^4 k^8 \alpha v c + 4 \Omega Q V^2 k^2 Q^2 k^4 v c \alpha + Q^2 k^8 V^4 v c \alpha \right) \right\}.$$

