<u>***Original Research Article***</u>

The Effect of Suspended Particles on Magneto-Gravitational Instability under the Influence of Electrical Resistivity

ABSTRACT

The problem of magneto-gravitational instability of rotating viscous, electrical conducting medium in the presence of suspended particles is studied Incorporating, thermal conductivity, and radiative heat-loss function. The Normal mode analysis is applied to derive the dispersion relation and it is discussed for wave propagation in longitudinal and transverse direction. Applying Routh-Hurwitz criterion the stability of the medium is discussed. The effect of suspended particles, magnetic field, rotation, resistivity and viscosity, Jean's criterion determines the condition of gravitational instability of gas particle medium. From the curves, we find that the effect of suspended particles, viscosity and temperature dependent heat-loss function have a stabilizing effect while density dependent heat-loss function has a destabilizing influence on the growth rate of an instability.

Key words: *Rotation, Finite Electrical Resistivity, Radiation, Suspended Particles, and Thermal Conductivity.*

PACS Nos. : - ***94.30.cq*****, 52.25.Xz, 94.05.Dd**

1. INTRODUCTION

The fragmentation of interstellar matter is a vitally important phenomenon in star formation. The gravitational instability of an infinite homogeneous self-gravitating gas was first investigated by Jeans (1902). Latter, in view of existence interstellar magnetic field Chandrasekhar (1961) has re-analyzed the same problem. Jeans (1929) analyzed the gravitational instability in an infinite homogenous medium, which derived the expression for maximum size of a uniform gravitating mass which is stable to small fluctuations in density.

In the past few years, the problem of magneto-gravitational instability of interstellar is of considerable importance in connection with protostar and star formation in magnetic dusty clouds. Magnetic field can provide pressure support and inhibit the contraction and fragmentation of interstellar clouds. Langer (1978) has investigated the stability of interstellar clouds against gravitational collapse and fragmentation in the presence of magnetic field. The problem of star formation in clouds containing magnetic field has been analyzed by Mestel and Spitzer (1956) and they have derived a stability criterion, in the form of a Jeans length for collapse based on the virial theorem. The nature of the coupling of the magnetic field to the neutrals through ion-neutral collisions has been analyzed by Spitzer (1962, 1968). A group of authors led by Sharma (1975, 1977 and 1980) has dealt with various problems of fluid dynamics in presence of suspended particles considering the effect of suspended particles on the onset of Benard convection, gravitational and magneto-gravitational instabilities of an infinite conducting homogeneous medium. Chhajlani *et al.* (1978) investigated the effect of finite conductivity on magneto-gravitational instability and suspended particles of a homogeneous medium. The problem of suspended particles and gravitational instability in rotating magnetized medium is investigated by Chhajlani and sanghavi (1985). Scanlon and Segel (1973) has investigated the effect of suspended particles on the instability of an infinite homogeneous gas-particle medium and further extended the problem to include the effect of a magnetic field. Sharma (1982) have incorporated Hall Current in the analysis of a self-gravitating magnetized gas-particle medium.

Along with this the rotation has also played an important role in the theory of star formation and fragmentation, of the protostellar clouds, to occur. It may be the primary cause of the same. Bondyopadhya (1972) has shown that the heating of the stellar interiors is influenced by the rotation, through damping of MHD waves. The assumption of infinite electrical conductivity for a fluid may yield serious discrepancies between theoretical predictions of idealized MHD and experiments. The problem of gravitational instability with finite conductivity, along with other parameters, has been worked out by Kossacki (1961), and Nayyar (1961). Raghavachar (1979) has studied the effect of a rotation on an unmagnetized gas-particle medium with suspended particles together with these studies.

In this way, thermal and radiative effects play an important role in the stability investigations and the interstellar medium. Field (1965) suggested that the observed filamentary condensations in nebulae may be due to thermal effects. The effect of thermal conductivity on magneto-gravitational instability incorporating different parameters has been studied by several authors as Vyas and Chhajlani (1987), Chhajlani and Vaghela (1987), Vaghela and Chhajlani (1987, 1989) and Chhajlani and Parihar (1993). Aggrawal and Talwar (1969) have investigated the problems of magneto thermal instability having heat-loss function. Bora and Talwar (1993) have studied the problem of thermal instability, having bearing on the formation of astrophysical condensations for a hydromagnetic field. The thermal instability in a star-gas system has been investigated by Talwar and Bora (1995). The radiative heat-loss functions are similar to those of cooling functions considered by Wolfire *et al.* (1995) and Shadmehri & Dib (2009) for the HII region. It leads to several important phenomena of astrophysics and space plasma physics. Kim and Narayan (2003) have investigated the thermal instability in clusters of galaxies with conduction taking on the role of the effect of radiative heat-loss function. Recently, Prajapati *et al.* (2010) have discussed radiative instability problem for self-gravitating rotating Hall plasma considering the effect of electron inertia, while, Kaothekar and Chhajlani (2012) corrected the radiative instability criterion with finite larmor radius corrections for non rotating viscous medium. Stiele *et al.* (2006) have discussed clump formation due to thermal instability in weakly ionized plasma. Inutsuka *et al.* (2005) have studied the propagation of shock waves into a warm neutral medium taking into account of radiative heating and cooling, thermal conduction and viscosity terms.

In all the above mentioned studies none of the author considers the simultaneous effects of thermal conductivity, radiative heat-loss function, rotation, viscosity, finite electrical resistivity and suspended dust particles on magneto-gravitational instability of infinite homogeneous plasma. In view of the importance of radiative effects in astrophysical contexts, we have incorporated radiative and thermal effects in the investigation of magneto-gravitational instability of an infinite homogeneous viscous, electrical conducting and rotating plasma in the presence of suspended dust particle. This aspect forms the subject matter of the present study. Although, the present treatment is also highly idealized but nevertheless of importance as it may be helpful in gaining an insight into the phenomenon of the gravitational instability.

2. EQUATION OF THE PROBLEM

We consider an infinite homogeneous self-gravitating, viscous, finite electrical conducting rotating plasma, composed of gas and the suspended particles in the presence of uniform magnetic field, thermal conductivity and radiative heat-loss function. The equation of the problem, with these effect, are written as

$$\rho\frac{D\vec{u}}{Dt} = -\vec{\nabla}p + \rho\vec{\nabla}U + \frac{1}{4\pi}\left(\vec{\nabla}\times\vec{B}\right)\times\vec{B} + \rho\nu\nabla^2\vec{u} + K_s N(\vec{v}-\vec{u}) + 2\rho\left(\vec{\Omega}\times\vec{u}\right) \quad (1)$$

$$\nabla^2 U = -4\pi G\rho \quad (2)$$

$$\frac{1}{(\gamma-1)}\frac{D}{Dt}\rho - \frac{\gamma}{(\gamma-1)}\frac{p}{\rho}\frac{D}{Dt}\rho + \rho\mathcal{L} - \lambda\nabla^2 T = 0 \quad (3)$$

$$\frac{D\rho}{Dt} + \rho\vec{\nabla}.\vec{u} = 0 \quad (4)$$

$$\left(\tau\frac{D}{Dt} + 1\right)\vec{v} = \vec{u} \quad (5)$$

$$p = \rho RT \quad (6)$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left(\vec{\mathrm{v}} \times \vec{B}\right) + \eta \nabla^2 \vec{B} \tag{7}$$

$$\vec{\nabla}.\vec{B} = 0 \tag{8}$$

where the parameters N, ρ, u, v, G, λ, p, T, U, R, γ, indicate the number density of particles, density of ionized component, fluid velocity, the particle velocity, gravitational constant, coefficient of thermal conductivity, fluid pressure, temperature, gravitational potential, gas constant and ratio of two specific heats, respectively. $\tau = m/K$ and mN is the mass of particles per unit volume. Operator $\frac{D}{Dt}$ is the substantial derivative given by

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{\mathrm{u}}\,.\vec{\nabla} \tag{9}$$

If we assume uniform particle size, spherical shape and small relative velocities between the two phases, then the net effect of particles on the gas is equivalent to an extra body force term per unit volume $K_s N$ (v–u) and is added to the momentum transfer equation for the gas, where the constant K_s is given by Stokes's drag formula K_s = 6□□νr, r being the particle radius and ν is the kinetic viscosity of clean gas. Inter-particles relations are also ignored by assuming the distance between particles to be too large compared with their diameters. The stability of the system is investigated by writing the solutions to the full equations as initial state plus a perturbation. We neglect the buoyancy force as its stabilizing effect for the case of two free boundaries is extremely small. The initial state of the system is taken to be a quiescent layer with a uniform particle distribution. The equations thus obtained are linearized by neglecting the product of two perturbed quantities. The perturbation in fluid velocity, pressure, density, magnetic field, gravitational potential, temperature, and the heat-loss function are given as $\vec{\mathrm{u}}$ (u_x, u_y, u_z), δp, $\delta\rho$, $\overrightarrow{B_1}$ (B_{1x}, B_{1y}, B_{1z}), δU, δT and $\delta\mathcal{L}$ respectively. The perturbation state is given as

$$\rho = \rho_0 + \delta\rho, \quad p = p_0 + \delta p, \quad \vec{B} = \vec{B}_0 + \delta \mathrm{B}, \ \mathrm{T} = T_0 + \delta T, \quad U = U_0 + \delta U,$$

$$\mathrm{u} = \mathrm{u}_0 + \delta\mathrm{u}, \text{ (with } \mathrm{u}_0 = 0), \ \mathrm{v} = \mathrm{v}_0 + \delta\mathrm{v}, \text{ (with } \mathrm{v}_0 = 0) \ \ \mathcal{L} = \mathcal{L}_0 + \delta\mathcal{L}, \text{ (with } \mathcal{L}_0 = 0) \tag{10}$$

Substituting equation (9) and (10) in all the above equation and then linearize them by neglecting the higher-order perturbations. Suffix '0' is dropped from the equilibrium quantities.

3. LINEARIZED PERTURBATION EQUATIONS:-

The linearized perturbation equations for self-gravitating viscous rotating medium in the presence of magnetic field and suspended particle incorporating thermal and electrical conductivity, and radiative effects are given as

$$\rho \frac{\partial \vec{\mathrm{u}}}{\partial t} = -\vec{\nabla}\delta p + \rho\vec{\nabla}\delta U + \frac{1}{4\pi}\left(\nabla \times \overrightarrow{B_1}\right) \times \overrightarrow{B_0} + \rho\nu\nabla^2\vec{\mathrm{u}} + K_s N(\vec{\mathrm{v}} - \vec{\mathrm{u}}) + 2\rho(\Omega \times \vec{\mathrm{u}}) \tag{11}$$

$$\nabla^2 \delta U = -4\pi G \delta\rho \tag{12}$$

$$\frac{1}{(\gamma-1)}\frac{\partial}{\partial t}\delta\rho - \frac{\gamma}{(\gamma-1)}\frac{p}{\rho}\frac{\partial}{\partial t}\delta\rho + \rho\left[\left(\frac{\partial\mathcal{L}}{\partial\rho}\right)_{\mathrm{T}}\delta\rho + \left(\frac{\partial\mathcal{L}}{\partial T}\right)_{\rho}\delta\mathrm{T}\right] - \lambda\nabla^2\delta\mathrm{T} = 0 \tag{13}$$

$$\frac{\partial\delta\rho}{\partial t} = -\rho\vec{\nabla}\vec{\mathrm{u}} \tag{14}$$

$$\left(\tau\frac{\partial}{\partial t} + 1\right)\vec{\mathrm{v}} = \vec{\mathrm{u}} \tag{15}$$

$$\frac{\delta p}{p} = \frac{\delta T}{T} + \frac{\delta\rho}{\rho} \tag{16}$$

$$\frac{\partial\overrightarrow{B_1}}{\partial t} = \vec{\nabla}\times\left(\vec{\mathrm{v}}\times\overrightarrow{B_0}\right) + \eta\nabla^2 B_1 \tag{17}$$

$$\vec{\nabla}.\overrightarrow{B_1} = 0 \tag{18}$$

4. DISPERSION RELATION

Let the perturbations of all quantities very as,

$$exp\,\{i(k_x x + k_z z + \sigma t)\} \tag{19}$$

where $k_{x,}$, k_z are the wave number of perturbation along the x and z axes and σ is the frequency of harmonic disturbances, such that $k_x^2 + k_z^2 = k^2$ combining equation (13) and (16), we obtain the expression for *δp* as

$$\delta p\left\{\omega + (\gamma-1)\left(\frac{\mathcal{L}_T T\rho}{p} + \frac{\lambda k^2 T}{p}\right)\right\} = \delta\rho\left\{\omega C^2 + (\gamma-1)\left(\mathcal{L}_T T - \mathcal{L}_\rho\rho + \frac{\lambda k^2 T}{\rho}\right)\right\} \tag{20}$$

Combining equation (12) – (20) in equation (11), we may write the following algebraic equations for the amplitude components. Here $\mathcal{L}_\rho$ and $\mathcal{L}_T$ respectively denote partial derivatives $(\delta\mathcal{L}/\delta\rho)_T$ and $(\delta\mathcal{L}/\delta T)_\rho$ of heat-loss function evaluated for the initial state.

$$\left(A + \frac{k^2V^2}{\omega+\eta k^2}\right)\mathrm{u}_x - 2\Omega_z\mathrm{u}_y + \frac{ik_x}{k^2}\Omega_{\mathrm{T}}^2\mathrm{s} = 0 \tag{21}$$

$$2\Omega_z\mathrm{u}_x + \left(A + \frac{k_z^2V^2}{\omega+\eta k^2}\right)\mathrm{u}_y - 2\Omega_x\mathrm{u}_z = 0 \tag{22}$$

$$2\Omega_x\mathrm{u}_y + A\mathrm{u}_z + \frac{ik_z}{k^2}\Omega_{\mathrm{T}}^2\, s = 0 \tag{23}$$

By taking the divergence of equation (11) and then combining it with equation (12) – (20), we obtain

$$\left(\frac{ik_xk^2V^2}{\omega+\eta k^2}\right)\mathrm{u}_x - [2i(k_x\Omega_z - k_z\Omega_x)]\mathrm{u}_y - (A\omega + \Omega_T^2)s = 0 \tag{24}$$

where $s = \frac{\delta\rho}{\rho}$ is the condensation of the medium, $V = \frac{B}{(4\pi\rho)^{1/2}}$ is the Alfven velocity, ω = iσ growth rate of instability, $c^2 = \gamma c'^2$, here c and $c'^2 = (p/\rho)^{1/2}$are the adiabatic and isothermal velocities of sound. We have made following substitution

$$\Omega_T^2 = \left(\frac{\omega\Omega_j^2+\Omega_I^2}{\omega+\beta}\right),\ \Omega_I^2 = (k^2\alpha - 4\pi G\rho\beta),\ \ \Omega_j^2 = (k^2c^2 - 4\pi G\rho),\ \ \Omega_m = \eta k^2,\ \ A = \frac{\tau\omega^2+\omega\xi+k^2\nu}{\omega\tau+1}$$

$$\xi = \left(1+\ \tau k^2\nu + \frac{K_s N\tau}{\rho}\right),\ \alpha = (\gamma-1)\left(\mathcal{L}_T T - \mathcal{L}_\rho \rho + \frac{\lambda k^2 T}{\rho}\right),\ \beta = (\gamma-1)\left(\frac{\mathcal{L}_T T\rho}{p} + \frac{\lambda k^2 T}{p}\right) \quad (25)$$

Equation (22) – (24) can be written in the following from:

$$\begin{vmatrix} \left(A+\dfrac{k^2V^2}{\omega+\Omega_m}\right) & -2\Omega_z & 0 & \dfrac{ik_x}{k^2}\Omega_T^2 \\ 2\Omega_z & \left(A+\dfrac{k_z^2V^2}{\omega+\Omega_m}\right) & -2\Omega_x & 0 \\ 0 & 2\Omega_x & A & \dfrac{ik_z}{k^2}\Omega_T^2 \\ \left(\dfrac{ik_xk^2V^2}{\omega+\Omega_m}\right) & -2i\left(k_x\Omega_z - k_z\Omega_x\right) & 0 & -(A\omega+\Omega_T^2) \end{vmatrix} \begin{vmatrix} u_x \\ u_y \\ u_z \\ s \end{vmatrix} = 0 \qquad (26)$$

For a non trivial solution of the Equation (26), the determinant of the matrix on the left-hand side should vanish, leading to the characteristic equation

$$\left[-A\left(A+\frac{k^2V^2}{\omega+\Omega_m}\right)\left(A+\frac{k_z^2V^2}{\omega+\Omega_m}\right)(\omega A+\Omega_T^2) - \left(A+\frac{k^2V^2}{\omega+\Omega_m}\right)(\omega A+\Omega_T^2)4\Omega_x^2 - A(\omega A +\right.$$

$$\Omega T24\Omega z2-kzk2A+k2V2\omega+\Omega m4\Omega x2\Omega T2\,2kx\Omega z-kz\Omega x+V2\omega+\Omega m4\Omega z\Omega xkxkz\Omega T2+2$$

$$kxk2\Omega T2A\Omega zkx\Omega z-kz\Omega x+kx2V2\omega+\Omega m\Omega T2A+kz2V2\omega+\Omega mA+4\Omega x2=0$$

(27)

This is the general dispersion relation (27) representing an infinite homogeneous plasma influenced by the effect of thermal conductivity, radiative heat-loss functions, rotation, viscosity, and finite electrical resistivity in the presence of suspended particles and uniform magnetic field.

On neglecting the effects of radiative heat loss function, thermal conductivity, and finite electrical resistivity, from above dispersion relation, we get the result of Chhajlani and Sanghavi (1985). In the absence of rotation and radiative heat-loss functions this dispersion relation reduces to Chhajlani and Parihar (1993) with Hall parameter taken as unity. Also in the absence of suspended particles and rotation this equation (27) reduces to Kaothekar and Chhajlani (2012) excluding the finite larmor radius and permeability.

Thus with these corrections we find that the dispersion relation (27) is modified due the combined effects of viscosity, finite electrical resistivity, thermal conductivity, rotation, arbitrary radiative heat-loss functions and suspended particles. This dispersion relation will be able to predict the complete information about the waves and instabilities of the radiative plasma considered in the presence of suspended particles.

5. DISCUSSION

We find that the general dispersion relation (27) is cumbersome for discussion; hence we shall now discuss the dispersion relation, by reducing it, for two special cases of interest. For the longitudinal and transverse mode of propagation, i. e., along the x and z axes, respectively.

5.1. Longitudinal Mode of Propagation

In this case the perturbation are taken parallel to the direction of the magnetic field i.e. ***($k_z = k$, $k_x = 0$)***. This is the dispersion relation reduces in the simple form to give

$$\left[-A(\omega A+\Omega_T^2)\left(A+\frac{k^2V^2}{(\omega+\Omega_m)}\right)^2-4\left(A+\frac{k^2V^2}{(\omega+\Omega_m)}\right)(\omega A+\Omega_T^2)\Omega_x^2-4\Omega^2(\omega A+\Omega_T^2)A+\right.$$

4Ωx2ΩT2A+k2V2ω+Ωm=0

(28)

The above dispersion relation shows the combined influence of thermal conductivity, radiative heat-loss functions, self-gravitation and finite electrical resistivity on rotating viscous uniformly magnetized plasma in the presence of suspended dust particles. The above equation is identical to Bora and Talwar (1993), when the effects of suspended particles, viscosity and rotation are not considered, excluding Hall current and finite electron inertia.

5.1.1. <u>Axis of rotation parallel to the magnetic field</u>

When $\Omega_x = 0$ and $\Omega_z = \Omega$, the dispersion relation (28) given as

$$(\tau\omega^2+\omega\xi+k^2\nu)(\omega A+\Omega_T^2)[(\omega+\Omega_m)\{(\tau\omega^2+\omega\xi+k^2\nu)^2+4\Omega_z^2(\omega\tau+1)^2\}+$$

V2k2ωτ+12+2V2k2ωτ+1τω2+ωξ+ k2νω+Ωm=0 (29)

The above dispersion relation shows the combined influence of thermal conductivity, arbitrary radiative heat-loss functions, gravitating attraction, rotating viscous plasma, uniformly magnetized, and finite electrical resistivity in the presence of suspended dust particles. This dispersion relation (29) is similar to Chhajlani and Sanghavi (1985) in the absence of radiative heat-loss function, thermal conductivity, and finite electrical resistivity. The dispersion relation (29) has three different components and our aim is to take out the physics involved in each component. So we discuss each component separately. The first component of the dispersion relation (29) gives

$$\omega^2\tau+\omega\xi+k^2\nu=0 \qquad (30)$$

The dispersion relation (30) represents a viscous mode combined with the effect of suspended particle which does not involve effect of rotation, gravitational attraction, radiative heat-loss function, thermal conductivity, electrical resistivity, and magnetic field. The above equation (30) is identical to equation (17) of Sharma (1977). Equation (30) does not admit any positive root or a complex whose real part is positive in this respect the system is said to be stable. It means that the viscosity of the medium together with suspended particle maintains the equilibrium of the system. The second factor of dispersion relation (29) equating to zero and on re-substituting the values of Ω_T^2 and A, we obtain the thermal gravitating mode of the form

$$\omega^4\tau + \omega^3\left(1 + \tau k^2\nu + \frac{K_sN\tau}{\rho} + \tau\beta\right) + \omega^2\left[k^2\nu + \left(1 + \tau k^2\nu + \frac{K_sN\tau}{\rho}\right)\beta + \Omega_{\mathrm{j}}^2\tau\right] +$$

$$\omega\left[\left(1 + \tau k^2\nu + \frac{K_sN\tau}{\rho}\right)k^2\nu\beta + \Omega_{\mathrm{j}}^2 + \Omega_{\mathrm{I}}^2\tau\right] + \Omega_{\mathrm{I}}^2 = 0 \qquad (31)$$

The above dispersion relation is a radiative conduction mode influenced by self-gravitation, viscosity and suspended particles. This mode is independent of finite electrical resistivity, rotation and magnetic field. Thus the condition of radiative instability will be unaffected by the presence of these parameters in the present case. To investigate the effect of various parameters, we discuss the dispersion relation (31) by reducing it to different cases of our interest.

For non-radiating ($\mathcal{L}_{\rho,T} = 0$) and thermally non-conducting, ($\lambda = 0$) medium the above dispersion relation (31) reduced to

$$\omega^3\tau + \omega^2\left(1 + \tau k^2\nu + \frac{K_sN\tau}{\rho}\right) + \omega\left[k^2\nu + \Omega_{\mathrm{j}}^2\tau\right] + \Omega_{\mathrm{j}}^2 = 0 \qquad (32)$$

The condition of instability is $\Omega_{\mathrm{j}}^2 < 0$, as given by Jeans. Again in the absence of suspended particle and viscosity we get the reduced form of eqution of (32) as

$$\omega^2 + \Omega_{\mathrm{j}}^2 = 0 \qquad (33)$$

This is the usual dispersion relation as dissuced by Chandrashekhar (1961). From the above equation again we get the The condition of instability is

$$\Omega_{\mathrm{j}}^2 = (k^2c^2 - 4\pi G\rho) < 0. \qquad (34)$$

Thus on comparing equation (32) and (33) it can be interpreted that suspended particles and viscosity does not alter the condition of instability and the Jeans criterion for gravitational instability in both cases is $k < k_j$, where $k_j = \sqrt{4\pi G\rho/c^2}$, denots the critical Jeans wave number. In the absence of suspended particles, dispersion relation (31) reduces to

$$\omega^3 + \omega^2[k^2\nu + \beta] + \omega\left[\beta k^2\nu + \Omega_{\mathrm{j}}^2\right] + \Omega_{\mathrm{I}}^2 = 0. \qquad (35)$$

This dispersion relation (35) is identical to Kaothekar and Chhajlani (2012). From equation (35), it is clear that the system will leads to the instability, when

$$k^2(\gamma - 1)\left(\mathcal{L}_TT - \mathcal{L}_\rho\rho + \frac{\lambda k^2T}{\rho}\right) - 4\pi G\rho(\gamma - 1)\left(\frac{\mathcal{L}_TT\rho}{p} + \frac{\lambda k^2T}{p}\right) < 0. \qquad (36)$$

This condition of instability (36) is same as the condition of radiative instability earlier obtained by Bora and Talwar (1993) and also Kaothekar and Chhajlani (2012). Now on comparing equation (31) and (35) we notice that in both cases the constant term is independent of the effect of suspended particles. It means that the presence of suspended particle does not contribute in the condition of instability and the criterion of radiative instability is unaffected by the presence of suspended particles. In the absence of self-gravitation the condition of instability (36) reduces to

$$\left(\mathcal{L}_T T - \mathcal{L}_\rho \rho + \frac{\lambda k^2 T}{\rho}\right) < 0. \tag{37}$$

This condition of instability is identical to the condition of thermal instability obtained by Field (1965). Thus with these comparisons of equation (34), (36) and (37), we can say that the condition of radiative instability is the combination of the condition of gravitational instability given by Jeans (1902) and the condition of thermal instability given by Field (1965). It means that the condition of radiative instability (36) is the modified condition of Gravitational instability in view of thermal and radiative effect and also the modified condition of thermal instability for self-gravitating medium.

We can write the dispersion relation (31) in non dimensional form in terms of self gravitation as

$$\bar{\varpi}^4 + \bar{\varpi}^3\left(\frac{1}{\tau} + \bar{\nu}\bar{k}^2 + \bar{\beta} + \bar{K}\right) + \bar{\varpi}^2\left[\frac{\bar{\beta}}{\tau} + \frac{\bar{\nu}\bar{k}^2}{\tau} + \bar{\beta}\bar{K} + \bar{\beta}\bar{\nu}\bar{k}^2 + \left(\bar{k}^2 - 1\right)\right] +$$

$$\bar{\varpi}\left[\frac{\left(\bar{k}^2-1\right)}{\tau} + \frac{\bar{\nu}\bar{k}^2\bar{\beta}}{\tau} + \left(\bar{k}^2\bar{\alpha} - \bar{\beta}\right)\right] + \frac{\left(\bar{k}^2\bar{\alpha} - \bar{\beta}\right)}{\tau} = 0 \tag{38}$$

where the various non dimensional parameters are defined

$$\bar{\lambda} = \frac{(\gamma-1)T\lambda\sqrt{4\pi G\rho}}{pc^2}, \quad \overline{\mathcal{L}_\rho} = \frac{(\gamma-1)\rho\mathcal{L}_\rho}{c^2\sqrt{4\pi G\rho}}, \quad \overline{\mathcal{L}_T} = \frac{(\gamma-1)\rho T\mathcal{L}_T}{p\sqrt{4\pi G\rho}}, \quad \bar{\alpha} = \frac{1}{\gamma}\left(\overline{\mathcal{L}_T} + \bar{\lambda}\bar{k}^2\right) - \overline{\mathcal{L}_\rho},$$

$$\bar{\beta} = \overline{\mathcal{L}_T} + \bar{\lambda}\bar{k}^2, \quad \bar{k} = \frac{kc}{\sqrt{4\pi G\rho}}, \quad \bar{K} = \frac{NK}{\rho\sqrt{4\pi G\rho}} \quad \bar{\varpi} = \frac{\omega}{\sqrt{4\pi G\rho}}, \quad \bar{\nu} = \frac{\nu\sqrt{4\pi G\rho}}{c^2},$$

In fig. (1-4), we have depicted the non-dimensional growth rate versus non dimensional wave number for various arbitrary values of suspended particles, density dependent heat-loss function, viscosity, and temperature dependent heat-loss function.

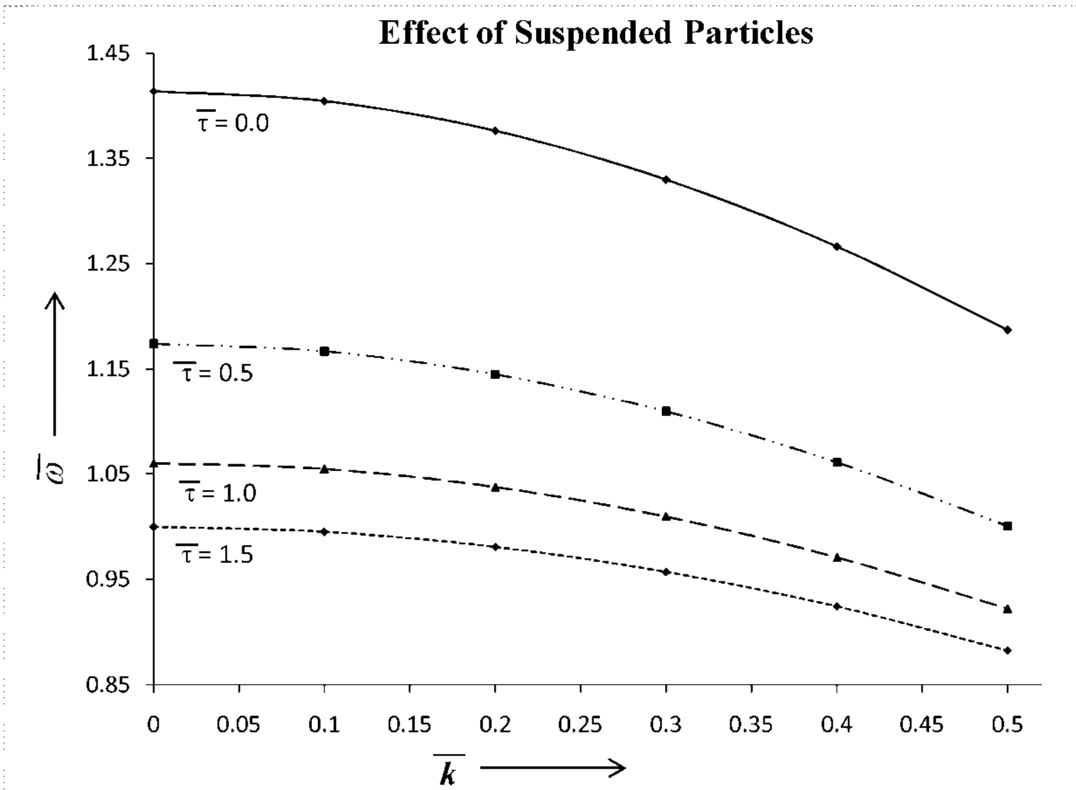

Figure-1: The growth rate is plotted against the non-dimensional wave number $\bar{k}$ with variation in the normalized values of relaxation time $\tau = 0.0, 0.5, 1.0, 1.5$ the value of other parameter are fixed $\bar{K} = \bar{\mathcal{L}}_T$= $\bar{\mathcal{L}}_\rho$= 0.5, and $\bar{\lambda}$ = $\bar{\nu}$ =1.0.

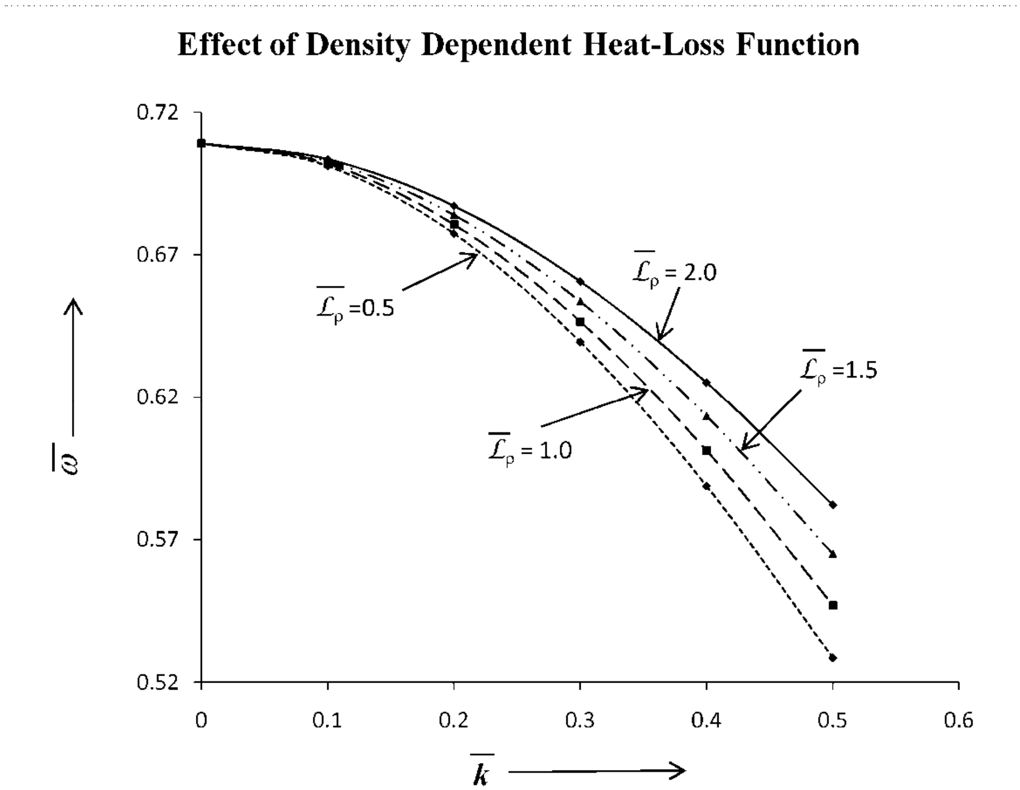

Figure-2: The growth rate is plotted against the non-dimensional wave number $\bar{k}$ with variation in the normalized values of density dependent radiative Heat-loss function $\bar{\mathcal{L}}_\rho$= 0.5, 1.0, 1.5, 2.0, the value of other parameter are fixed $\bar{\mathcal{L}}_T$= $\bar{K} =$ $\bar{\lambda}$ = $\bar{\nu}$ =1.5 and $\tau = 0.5$.

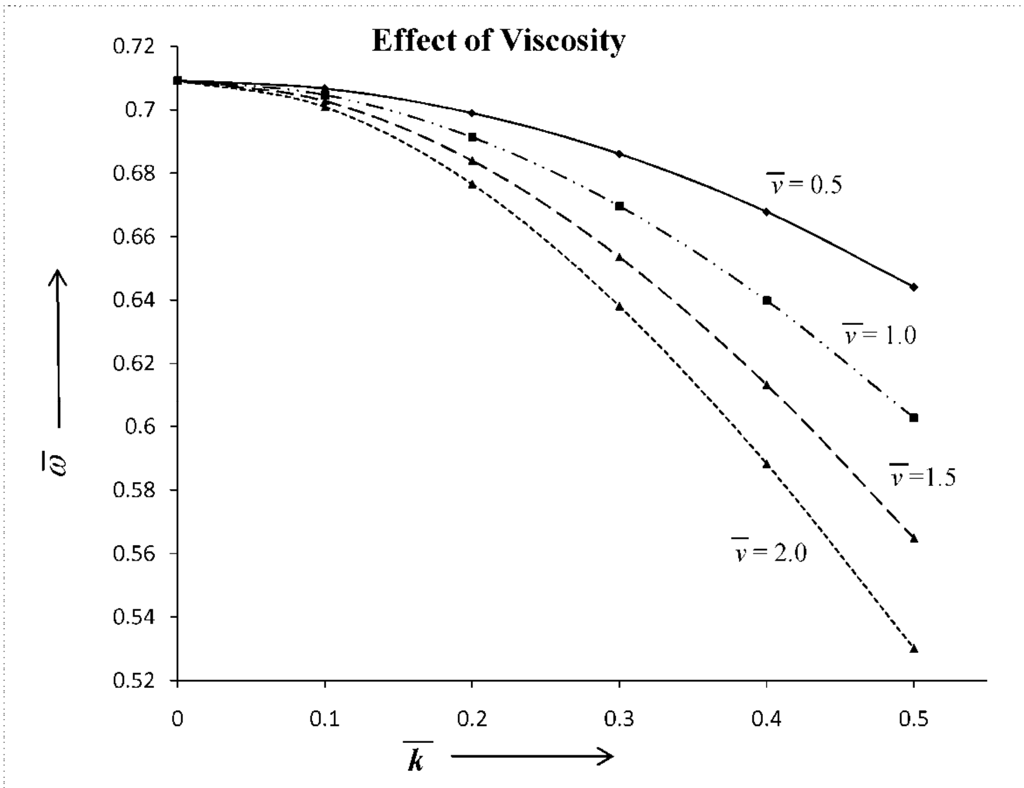

Figure-3: The growth rate is plotted against the non-dimensional wave number $\bar{k}$ with variation in the normalized values of viscosity $\bar{\nu}$ = 0.5, 1.0, 1.5, 2.0, the value of other parameter are fixed $\bar{\mathcal{L}}_{\rho}$= $\bar{\mathcal{L}}_T$ = $\bar{K} = \bar{\lambda}$ = 1.5 and $\tau = 0.5$

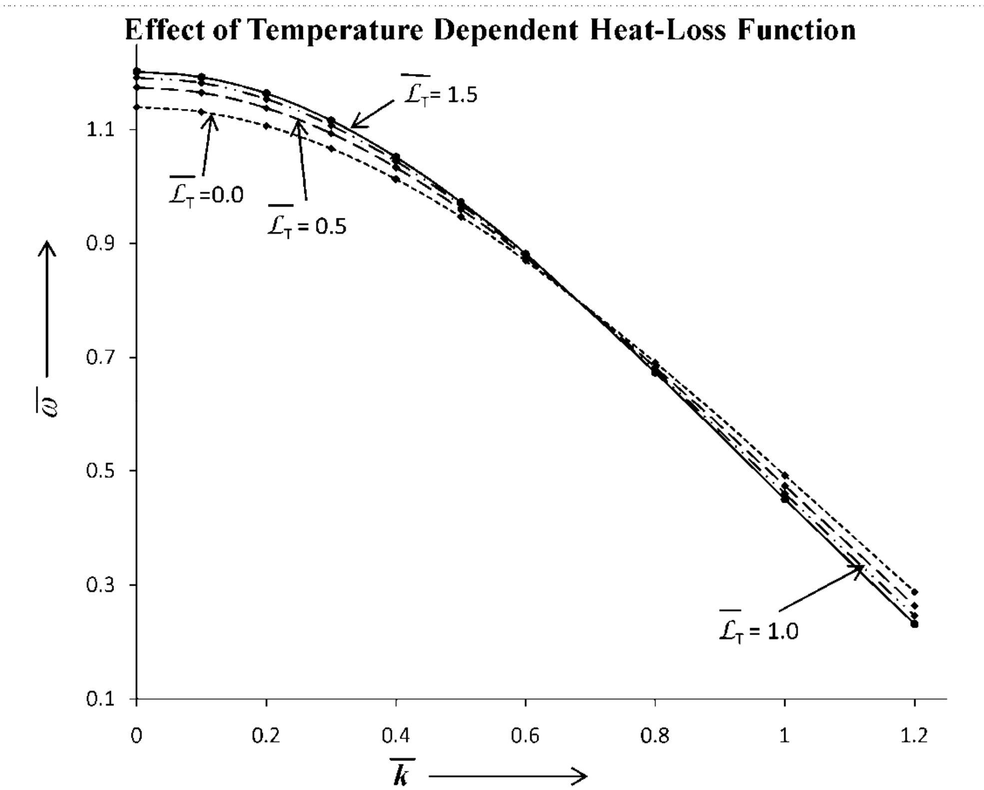

Figure-4: The growth rate is plotted against the non-dimensional wave number $\bar{k}$ with variation in the normalized values of temperature dependent radiative Heat-loss function $\bar{\mathcal{L}}_T$=0.0, 0.5, 1.0, 1.5, the value of other parameter are fixed $\bar{\mathcal{L}}_\rho$= $\bar{K} = \tau =$ 0.5 and $\bar{\lambda}$ = $\bar{\nu}$ =1.5.

In Fig.-1 we have plotted for the growth rate of an unstable mode (positive imaginary root of $\bar{\omega}$) against the non-dimensional wave length $\bar{k}$ with variation in the value of τ = (0.0, 0.5, 1.0, 1.5) with taking the values of other parameter are fixed $\bar{K} = \bar{\mathcal{L}}_T = \bar{\mathcal{L}}_\rho$= 0.5, and $\bar{\lambda}$ = $\bar{\nu}$ =1.0. We find that the growth rate of instability decreases with increase in the relaxation time τ. Hence, we can conclude that the system tends to be stabilize and maintain equlibrium in the presence of suspended particles.

Fig.-2 is plotted for the growth rate of an unstable mode (positive roots of $\bar{\omega}$) against the non-dimensional wave length $\bar{k}$ with variation in the density dependent radiative Heat-loss function $\bar{\mathcal{L}}_\rho$= 0.5, 1.0, 1.5, 2.0 with taking the value of $\bar{\mathcal{L}}_T$ = $\bar{K} =$ $\bar{\nu}$ =$\bar{\lambda}$= 1.5, and τ = 0.5. We find that the growth rate of instability also increases with increase in the density dependent radiative Heat-loss function $\bar{\mathcal{L}}_\rho$. Hence the the density dependent radiative Heat-loss function has a destabilizing influence on the growth rate of the instability.

Fig.-3 is plotted for the growth rate of an unstable mode (positive roots of $\bar{\omega}$) against the non-dimensional wave length $\bar{k}$ with variation in the viscosity $\bar{\nu}$ = 0.5, 1.0, 1.5, 2.0, with taking the value of $\bar{\mathcal{L}}_\rho$= $\bar{\mathcal{L}}_T$ = $\bar{K} = \bar{\lambda}$ = 1.5. and τ = 0.5. We find that the growth rate of instability decreases with increase in the viscosity $\bar{\nu}$. Hence the viscosity has a stabilizing influence on the growth rate of the instability.

Fig.-4 is plotted for the growth rate of an unstable mode (positive roots of $\bar{\omega}$) against the non-dimensional wave length $\bar{k}$ with variation in the temperature dependent radiative Heat-loss function $\bar{\mathcal{L}}_T$= 0.5, 1.0, 1.5, 2.0 with taking the value of $\bar{\mathcal{L}}_\rho = \bar{K} = \tau$ = 0.5, and $\bar{\lambda} = \bar{\nu}$ = 1.5. We find that the growth rate of instability also increases with decrease in the temperature dependent radiative Heat-loss function $\bar{\mathcal{L}}_T$. Hence the the temperature dependent radiative Heat-loss function has a stabilizing influence on the growth rate of the instability.

Now the third factor of equation (29) is equating to zero and after solving we obtain dispersion relation as

$$\omega^6\tau^2 + \omega^5[2\tau^2\Omega_m + 2\tau\xi] + \omega^4[\tau^2\Omega_m^2 + 4\tau\xi\Omega_m + 2\tau k^2\nu + \xi^2 + 4\tau^2\Omega^2 + 2V^2k^2\tau^2] +$$
$$\omega^3[2\tau\xi\Omega_m^2 + 4\tau k^2\nu\Omega_m + 2\xi k^2\nu + 2\xi^2\Omega_m + 8\tau\Omega^2 + 8\Omega_m\tau^2\Omega^2 + 2V^2k^2(\Omega_m\tau^2 + \tau +$$
ξτ+ω22τk2νΩm2+ξ2Ωm2+4ξk2νΩm+k2ν2+4Ω21+Ωm2τ2+4Ωmτ+V4k4τ2+2V2
k2Ωmτ+ξ+τk2ν+Ωmξτ+ω2ξk2νΩm2+2Ωmk2ν2+8Ω2Ωm2τ+Ωm+2V4k4τ+2V2k
2Ωmξ+k2ν+k2νΩmτ+k2ν2Ωm2+4Ωm2Ω2+V4k4+2V2k2Ωmk2ν=0

(39)

Equation (39) is a sixth-degree polynomial equation incorporating the simultaneous effects of viscosity, magnetic field, finite electrical conductivity, rotation and suspended particles. This dispersion relation shows the Alfven mode modifies due to these parameters. It is noted that the constant term of the equation (39) is always positive; thus there is no positive root and thereby the system is stable. In the absence of suspended particles $\tau = 0$, finite electrical conductivity $\Omega_m = 0$, equation (39) reduces to the form

$$\omega^4 + \omega^3[2k^2\nu] + \omega^2[(k^2\nu)^2 + 4\Omega^2 + 2V^2k^2] + \omega[2V^2k^2k^2\nu] + V^4k^4 = 0 \qquad (40)$$

which has been already discussed by Chandrasekhar (1961). Thus, we find that simple Alfven mode is modified in the form of equations (39), and (40), in the presence rotation, finite electrical conductivity, viscosity and suspended particles, respectively. Since all the coefficient of the polynomial are positive real numbers, applying necessary condition of Hurwitz's criterion we find that all the roots of above equation are either negative real complex with negative real parts. Hence all the roots correspond to stable modes. We calculate the minors and get

$$\Delta_1 = 2k^2\nu > 0. \quad \text{as} \quad \gamma > 1$$

$$\Delta_2 = 2(k^2\nu)^3 + 8k^2\nu\Omega^2 + 2k^2\nu \mathrm{V}^2 k^2 > 0.$$

$$\Delta_3 = 4(k^2\nu)^4 \mathrm{V}^2 k^2 + 16(k^2\nu)^2 \mathrm{V}^2 k^2 \Omega^2 > 0.$$

$$\Delta_4 = \mathrm{V}^4 k^4 \Delta_3 > 0.$$

These all Δ's are positive, thereby, satisfying the Routh-Hurwitz criterion, Hence, the system expressed by equation (40) is stable if $\Omega_{\mathrm{j}}^2 > 0$ and $\Omega_{\mathrm{I}}^2 > 0$.

5.1.2. <u>Axis of rotation perpendicular to the magnetic field</u>

When $\Omega_x = \Omega$, and $\Omega_z = 0$ in the dispersion relation(28) gives as

$$(\tau\omega^2 + \omega\xi + k^2\nu)\left[(\omega A + \Omega_T^2)\left(A + \frac{k^2V^2}{(\omega+\Omega_m)}\right)^2 + 4\Omega^2\sigma\left(A + \frac{k^2V^2}{(\omega+\Omega_m)}\right)\right] = 0 \qquad (41)$$

The dispersion relation (41) shows the combined influence of thermal conductivity, arbitrary radiative heat-loss functions, gravitating attraction, rotating viscous plasma, uniformly magnetized, and finite electrical resistivity in the presence of suspended dust particles. This dispersion relation (41) is the product of two independent factors. These factors show the mode of propagations incorporating different parameters as discussed below. The first factors of equation (41) is obtained

$$(\tau\omega^2 + \omega\xi + k^2\nu) = 0 \qquad (42)$$

This dispersion relation is same as equation (30), which has been discussed earlier. The second factor of (41) gives, on substituting the values of Ω_{T}^2 and A, the following tenth order polynomial equation

$$\omega^{10} + \omega^9 A_1 + \omega^8 A_2 + \omega^7 A_3 + \omega^6 A_4 + \omega^5 A_5 + \omega^4 A_6 + \omega^3 A_7 + \omega^2 A_8 + \omega A_9 + A_{10} = 0 \qquad (43)$$

$$A_{10} = [\Omega_I^2\{\Omega_m^2(k^2\nu)^2 + \mathrm{V}^2k^2(2\Omega_m k^2\nu + \mathrm{V}^2k^2)\}]$$

The dispersion relations (43) are very lengthy hence its constant term of the last coefficients gives the condition of instability. It represents the effect of finite electrical resistivity, thermal conductivity, arbitrary radiative heat-loss functions, viscosity, rotation, on the self gravitational instability of homogeneous plasma with axis of rotation perpendicular to the direction of magnetic field and with longitudinal mode of propagation. The condition of instability is obtained from constant term of equation (43) and gives as

$$\Omega_I^2 = k^2\alpha - 4\pi G\rho\beta < 0. \qquad (44)$$

The above condition of instability is identical to the condition (36) we find that the condition of instability for this mode of propagations, in both the cases of rotation perpendicular and parallel to the magnetic field, is the same. It means that there is no effect of the rotation and its direction, perpendicular and parallel to the magnetic field, on the condition of radiative instability.

5.2. Transverse Propagation

In this case the perturbations are taken perpendicular to the direction of the magnetic field. For, our convenience, we take $\boldsymbol{k_x = k}$, and $\boldsymbol{k_z = 0}$, the general dispersion relation (27) reduces to

$$-(\omega A + \Omega_T^2)\left[\left(A + \frac{k^2V^2}{(\omega+\Omega_m)}\right)\{A^2 + 4\Omega_x^2\} + 4\Omega_z^2 A\right] + \Omega_{\mathrm{T}}^2\left[4\Omega_z^2 A + \right.$$

$$k2V2\omega+\Omega mA2+4\Omega x2=0 \qquad (45)$$

We find that for transverse mode of propagation the dispersion relation is due to the presence of magnetic field, self gravitation, thermal conductivity, arbitrary radiative heat loss function, rotation, and finite electrical resistivity on the self-gravitational instability.

5.2.1. Axis of rotation along the magnetic field

When $\Omega_x = 0$ and $\Omega_z = \Omega$, the dispersion relation (45) gives as

$$A^2\left[A(\omega A + \Omega_T^2) + \frac{\omega k^2V^2}{(\omega+\Omega_m)}A + \omega 4\Omega_z^2\right] = 0 \qquad (46)$$

This equation represents the effect of thermal conductivity, magnetic field, radiative heat-loss function, rotation, gravitational attraction and finite electrical resistivity in the presence of suspended particles. Equation (46) has two independent factors, each representing a different mode of propagation. The first factor of equation (46) gives the following result on substituting the values of A, we get

$$(\tau\omega^2 + \omega\xi + \ k^2\nu) = 0 \qquad (47)$$

The first factor is identical to equation (30) and represents a viscous type damped mode modified due to the presence of suspended particles.

The second factor of equation (46) gives the following result on substituting the values of Ω_{T}^2, and A, we get

$$\omega^7\tau^2 + \omega^6[\Omega_{\mathrm{m}} + \beta\tau^2 + 2\tau\xi] +$$

$$\omega^5\left[\Omega_m\beta\tau^2 + 2\tau\xi\beta + 2\tau\xi\Omega_m + 2\tau k^2\nu + \xi + \Omega_j^2\tau^2 + \mathrm{V}^2k^2\tau^2 + 4\tau^2\Omega^2\right] +$$

$$\omega^4\left[2\tau\xi\Omega_m\beta + 2\tau k^2\nu\beta + \xi^2(\beta + \Omega_{\mathrm{m}}) + \Omega_I^2\tau^2 + \mathrm{V}^2k^2\tau^2\beta + (\tau^2\beta + \Omega_{\mathrm{m}}\tau^2 + 2\tau)4\Omega^2 + \right.$$

$$\mathrm{V}2k2\tau+\Omega j2\tau 1+\xi+2\tau k2\nu\Omega m+2k2\nu\xi+\tau 2\Omega m\Omega j2+$$

$$\omega^3\left[\Omega_m\beta(2\tau k^2\nu + \xi^2) + 2\xi k^2\nu(\beta + \Omega_m) + \Omega_{\mathrm{m}}\tau^2\Omega_I^2 + \left(\mathrm{V}^2k^2\tau\beta + \Omega_I^2\tau + \tau\Omega_m\Omega_j^2\right)(1 + \xi) + \right.$$

$$4\Omega 2\,1+\Omega m\tau 2\beta+2\tau\beta+\Omega m\tau+(k2\nu)2+\mathrm{V}2k2+\Omega j2\xi+k2\nu\tau+$$

$\omega^2\left[2\xi k^2\nu\Omega_m\beta + (k^2\nu)^2(\beta + \Omega_m) + \tau\Omega_m\Omega_I^2(1 + \xi) + (4\Omega^2)(\beta + 2\Omega_m\tau\beta + \Omega_m) +\right.$

V2𝑘2β+Ω𝐼2ξ+𝑘2𝜈τ+Ω𝑗2Ω𝑚ξ+𝑘2𝜈+τΩ𝑚𝑘2𝜈+V2𝑘2𝑘2𝜈+

$\omega\left[(k^2\nu)^2\Omega_m\beta + \Omega_I^2(\Omega_m\xi + k^2\nu + \Omega_m k^2\nu\tau) + \Omega_m\beta(4\Omega^2) + V^2k^2\Omega_v\beta + k^2\nu\Omega_m\Omega_j^2\right] +$

$$\Omega_m k^2\nu\Omega_I^2 = 0. \qquad (48)$$

Equation (48) represents the dispersion relation for transverse propagation through infinite homogeneous self-gravitating viscous magnetized plasma with, radiative effects, finite electrical resistivity, rotation and suspended dust particles. when $\Omega_I^2 < 0$, equation (48) has at least one real positive root which renders the system unstable. So the condition of instability for transverse mode of propagation is given as

$$[k^2\alpha - 4\pi G\rho\beta] < 0. \qquad (49)$$

This equation is same equation (36), we has been already discussed in earlier. we reduce the dispersion relation (48) for the infinitely conducting medium ($\Omega_m = 0$), we get

$\omega^6\tau^2 + \omega^5[\beta\tau^2 + 2\tau\xi] + \omega^4\left[2\tau\xi\beta + 2\tau k^2\nu + \xi + \Omega_j^2\tau^2 + V^2k^2\tau^2 + 4\tau^2\Omega^2\right] + \omega^3\left[2\tau k^2\nu\beta +\right.$

β𝜉2+Ω𝐼2τ2+V2𝑘2τ2β+𝜏2𝛽+2τ4Ω2+

V2𝑘2τ+Ω𝑗2𝜏1+ξ+2𝑘2𝜈ξ+𝜔22ξ𝑘2𝜈𝛽+V2𝑘2τβ+Ω𝐼2𝜏1+ξ+4Ω21+2τβ+(𝑘2𝜈)2+V2𝑘2+Ω𝑗2

ξ+𝑘2𝜈τ+𝜔(𝑘2𝜈)2𝛽+4Ω2𝛽+V2𝑘2β+Ω𝐼2ξ+𝑘2𝜈τ+Ω𝑗2𝑘2𝜈+V2𝑘2𝑘2𝜈+𝑘2𝜈ΩI2+V2𝑘2𝛽=0

(50)

Equation (50) represents the dispersion relation for transverse propagating through finite homogeneous, self-gravitating, viscous magnetized, thermal conductivity, rotation, and radiative effects with suspended dust particles. In this mode constant terms of dispersion relation (48) and (50) is independent of suspended particles. Thus, we can say that the effect of suspended particles does not contribute to determine the condition of instability but it changes the growth rate of instability of the system. The condition of instability for infinitely electrical conducting medium represented by (50) in transverse mode of propagation is given as

$$[k^2\alpha - 4\pi G\rho\beta + V^2k^2\beta] < 0 \qquad (51)$$

The above condition of instability is identical to that of obtained by Prajapati *et al.* (2010) for the direction of propagation, transverse to the magnetic field. The comparison equation (49) and (51) reveals that there is an in extra term, showing the effect of magnetic field, added in the condition of instability (51) for infinitely electrical conducting medium. It means that the effect of finite electrical resistivity eliminate the effect of magnetic field from the condition of instability. Again, in the absence of viscosity ($\nu = 0$) and suspended particles ($\tau = 0$), the dispersion relation (50) reduces to get

$$\omega^4 + \omega^3\beta + \omega^2\left[4\Omega^2 + V^2k^2 + \Omega_j^2\right] + (4\Omega^2 + V^2k^2)\beta + \Omega_I^2 = 0 \qquad (52)$$

The constant term of the above equation leads to the condition of instability if

$$k^2\alpha + (4\Omega^2 + \mathrm{V}^2 k^2 - 4\pi G\rho)\beta < 0. \tag{53}$$

And the corresponding wave number is given as

$$k_{j1} = \sqrt{\frac{M \pm [M^2 + N]^{1/2}}{2}}. \tag{54}$$

$$\text{where } M = \left[\left(\frac{\rho^2 \mathcal{L}_\rho}{\lambda T} + \frac{4\pi G\rho}{c'^2} - \frac{4\varepsilon\Omega^2}{c'^2}\right)\left(1 + \frac{\mathrm{v}^2}{\chi\, c'^2}\right)^{-1} - \frac{\rho \mathcal{L}_T}{\lambda}\right]. \quad N = \left[\frac{16\mathcal{L}_T}{\lambda\, c'^2}(\pi G\rho^2 - \Omega^2)\right].$$

On comparing equation (51) and (53) we can say that the absence of viscosity parameter adds an extra term, representing the effect of rotation, in the condition of radiative instability (51). From the above equation (54) we can say that the medium is unstable for all wave number $k < k_j$. Again we analyze that the inclusion of rotation term in equation (54) decreases the critical wave number and preserve the system stability. Thus we conclude that contribution of rotation in the condition of radiative instability is only observed when fluid is in-viscid.

If we compare the condition of instability (53) with that of obtained by Bora and Talwar (1993) we see that the condition of instability is modified due to the rotation and magnetic field for in-viscid and infinitely electrical conducting plasma. The growth of instability is decreased due to magnetic field and rotation of the system in the transverse mode of propagation. Thus the effect of magnetic field and rotation is to stabilize the system. We may also compare the situation with that which obtained by Field (1965) for the case of non-gravitational condensation phenomena in astronomy. With this comparison we find that the critical wave number for self-gravitating radiating system is remarkable different from that for non-gravitating radiating system. If we ignore the effect of rotation, viscosity, finite electrical resistivity, suspended particle and self-gravitation then we obtain the modified result of Field (1965) with a magnetic field. Again the result of present work is the modified result of Chandrashekhar (1961) for gravitational instability due to our consideration of thermally conducting and radiating medium.

5.2.2. <u>Axis of rotation perpendicular to the magnetic field</u>

When $\Omega_x = \Omega$, and $\Omega_z = 0$ in the dispersion relation (45) and this gives.

$$\mathrm{A}\,\{\mathrm{A}^2 + 4\Omega^2\}\left\{(\omega A + \Omega_{\mathrm{T}}^2) + \frac{\omega \mathrm{k}^2 \mathrm{V}^2}{(\omega + \Omega_m)}\right\} = 0 \tag{55}$$

Equation (55) represents the dispersion relation for transverse wave propagation when axis of rotation is perpendicular to magnetic field. This equation is dependent on the effect of radiative heat-loss function, self-gravitation, viscosity, finite electrical resistivity, thermal conductivity, magnetic field and rotation, in the presence of suspended dust particles. Equation (55) has three independent factors, each representing a different mode of propagation. The first factor of (55) is same as equation (30), which shows a viscous stable mode sustain by suspended particles. The second factor of equation (55) equating to zero, gives the following result

$$\omega^4\tau^2 + \omega^3[2\tau\xi] + \omega^2[\xi^2 + 2\tau k^2\nu + 4\Omega^2\tau^2] + \omega[2\xi k^2\nu + 4\Omega^2\tau] + (k^2\nu)^2 + 4\Omega^2 = 0 \quad (56)$$

This dispersion relation shows a separate rotating mode with the effect of suspended dust particle, and viscosity of the plasma medium, which is independent of thermal conductivity, finite electrical conductivity, and radiative heat loss function. This is also a stable damped mode, which represents a damping effect due to viscosity, rotation and suspended particles. The third factor of equation (55), on substituting the values of Ω_{T}^2, gives the following result

$$\omega^5\tau + \omega^4[\xi + \beta + \Omega_m] + \omega^3\left[k^2\nu + \xi(\beta + \Omega_m) + \tau k^2V^2 + \tau\Omega_j^2 + \tau\Omega_m\beta\right] +$$

$$\omega^2\left[\xi\Omega_m\beta + k^2\nu(\Omega_m + \beta) + \tau\Omega_I^2 + \Omega_j^2(1 + \tau\Omega_m) + k^2V^2(1 + \tau\beta)\right] +$$

$$\omega\left[k^2\nu\Omega_m\beta + \Omega_I^2(1 + \tau\Omega_m) + \Omega_j^2\Omega_m + k^2V^2\beta\right] + \Omega_I^2\Omega_m = 0 \quad (57)$$

This equation shows the effect of simultaneous inclusion of radiative heat-loss function, thermal conductivity, viscosity and finite electrical resistivity on the magneto-gravitational instability of the system in the presence of suspended particles. The constant term of this dispersion relation is the condition $\Omega_{\mathrm{I}}^2 < 0$ is satisfied, the above equation admits at least one real positive root which leads to the instability of the system. So the condition of instability for transverse mode of propagation is given as

$$\Omega_{\mathrm{I}}^2 = [k^2\alpha - 4\pi G\rho\beta] < 0 \quad (58)$$

This equation is same equation (36), we has been already discussed in earlier. In the absence of finite electrical resistivity ($\Omega_m = 0$), and suspended particles ($\tau = 0$) the system leads to the condition of instability when

$$[k^2\alpha - 4\pi G\rho\beta + \mathrm{V}^2k^2\beta] < 0 \quad (59)$$

and the expression for critical wave number is given as

$$2\left(1 + \frac{\mathrm{v}^2}{\mathrm{c}^2}\right)k_{j2}^2 = \left[\left\{\frac{4\pi G\rho}{c^2} + \frac{\rho^2\mathcal{L}_\rho}{\lambda T} - \frac{\rho\mathcal{L}_T}{\lambda}\left(1 + \frac{\mathrm{v}^2}{\mathrm{c}^2}\right)\right\} \pm \left\{\left(\frac{4\pi G\rho}{c^2} + \frac{\rho^2\mathcal{L}_\rho}{\lambda T} - \right.\right.\right.$$

$$\rho\mathcal{L}T\lambda 1{+}\mathrm{v}2\mathrm{c}2 2{+}16\pi G\rho 2\mathcal{L}T\lambda c2 1{+}\mathrm{v}2\mathrm{c}2 12 \quad (60)$$

This condition of instability (59) is same as equation (51), for transverse mode of propagation with axis of rotation is parallel to the magnetic field. Here we observe that a magnetic field has a stabilizing role in the transverse mode of propagation; the above parameters remarkably influence the growth in this mode of propagation. In the case of longitudinal mode of propagation, there is no effect of the magnetic field on the thermal mode. Thus, the magnetic field has no influence on the growth in the longitudinal mode of propagation but in the case of transverse mode of propagation, the magnetic field plays its role in the growth rate of the instability due to the presence in various coefficients in the dispersion relation. Again it is noted that in the transverse direction of propagation, rotation has a stabilizing influence on the growth rate of instability. In the transverse direction of propagation with axis of rotation is parallel to the magnetic field, radiative instability criteria is modified due to rotation for in-viscid fluid while in direction of rotation perpendicular to the magnetic field, rotation does not affect the instability criterion and affect the growth rate by showing separate mode in the dispersion relation.

6. CONCLUSION

To consider the main features of the present problem, we examine the self-gravitational instability of partially-ionized plasma under the influence of finite electrical resistivity, thermal conductivity, radiative heat-loss functions, magnetic field and suspended particles. The general dispersion relation is obtained, which is modified due to the presence of these parameters. This dispersion relation is reduced for longitudinal and transverse modes of propagation. We conclude that the condition of radiative instability remains valid but the expression of the critical wave number is modified. From the inequality of instability we see that magnetic field and rotation affect the critical Jeans wave number in transverse mode of propagation but do not affect the criterion of instabilities in longitudinal mode of propagation.

In the case of longitudinal propagation we obtain Alfven mode modified by the viscosity, electrical resistivity and rotation in the presence of suspended particles. The thermal conductivity has a destabilizing influence. We find that the condition of instability is unaffected by the presence of magnetic field, rotation, electrical resistivity in the presence of suspended particles and remain same as previous result. From the curves, it is also found that the density dependent heat-loss function has a destabilizing influence while temperature dependent heat-loss function has a stabilizing influence on the instability of the system. Also the viscosity and the presence of suspended particles stabilize the system.

In the transverse mode of propagation, we find a gravitating thermal mode influenced by thermal conductivity and arbitrary radiative heat-loss functions. We also found the condition of instability and the expression of critical Jeans wave number both are modified due to the presence of magnetic field, rotation, thermal conductivity and arbitrary radiative heat-loss function. The effect of rotation modifies the condition of instability for rotation parallel to the magnetic field when fluid is in-viscid and for rotation perpendicular to the magnetic field, it does not affect instability criterion.

REFERENCES

1. Aggrawal M, Talwar SP, Publ. Astron. Soc. Japan., 1969;21:176.
2. Bondyopadhya R, Czech. J. Phys. 1972;B22:1199.
3. Bora MP, Talwar SP, Phys. Fluids, 1993;B5(3):950.
4. Chandrasekhar S, “Hydrodynamic and Hydromagnetic Stability”, Clarendon Press, Oxford; 1961.
5. Chhajlani RK, Parihar, AK, Contrib. Plasma Phys. 1993;33:3.

6. Chhajlani RK, Sanghvi RK, Contrib. Plasma Phys. 1985;25(6):623.
7. Chhajlani RK, Vaghela DS, Astrophys. Space Sci., 1987;134:301.
8. Chhajlani RK, Vashistha P, Bhand SC, Z. Naturforsch, 1978;339:1469.
9. Field GB, Astrophysics J., 1965;142:265.
10. Inutsuka S, Koyama H, Inoue T, AIP Conf. Proc. 2005;784:318.
11. Jeans JH, Phil. Trans. Roy. Soc. London 1902;A199:1.
12. Jeans JH, "Astronomy and Cosmogony", Cambridge Uni. Press, Cambridge; 1929.
13. Kaothekar S, Chhajlani RK, Journal of Physics: conference series 2012;365:012044.
14. Kim W, Narayan R, Astrophys. J. 2003;596:L139.
15. Kossacki KM, Acta astron. 1961;11:83.
16. Langer WD, Astrophys. J. 1978;225:95.
17. Menou K, Balubs SA, Spruit HC, Astrophys. J. 2004;607:564.
18. Mestel L, Spitzer L, Mon. Not. Roy. Astron. Soc. 1956;116:503.
19. Nayar, NK, Z. Astrophys. 1961;52:266.
20. Prajapati RP, Pensia RK, Kaothekar S, Chhajlani RK, Astrophys. Space Sci. 2010;327:139.
21. Raghavachara MR, Phys. Fluids 1979;22:999.
22. Scanlon JW, Segel LA, Phys. Fluids 1973;16:1573.
23. Shadmehari M, Dib S, Mon. Not. Roy. Astron Soc. 2009;395:985.
24. Sharma KC, Astrophysics Space Sc., 1982;85:263.
25. Sharma RC, Sharma KC, Astrophys. Space Sci. 1980;71:325.
26. Sharma RC, Zeit Apple. Math. Mech. 1975;55:615.
27. Sharma RC, Astrophys. Space Sci. 1977;46:255.
28. Spitzer L, "Physics of Fully Ionized Gases", Interscience, New York;1962.
29. Spitzer L,"Diffuse Matter in space", Interscience, New York;1968.
30. Stiele H, Lesch H, Heitsch F, Mon. Not. Roy. Astron. Soc. 2006;372:862.
31. Talwar SP, Bora MP, J. Plasma Phys. 1995;54:157.
32. Vaghela DS, Chhajlani RK, Contrib. Plasma Phys.1987;27:3.
33. Vaghela DS, Chhajlani RK, Contrib. Plasma Phys. 1989;29:1.
34. Vyas MK, Chhajlani, RK, Astrophys. Space Sci. 1987;140:89.
35. Wolfire MG, Hollenbach D, McKee CF, Tilens AGGM, Baker ELO, Astrophys. J. 1995;443:15.