Effect of Channel Slope on Hydraulic Jump Characteristics

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ABSTRACT

Hydraulic jump mainly serves as an energy dissipater to dissipate excess energy of flowing water downstream of hydraulic structures, such as spillway, sluice gates etc. This research paper investigates the effect of channel slope on the characteristics of free hydraulic jump and the energy dissipation downstream the gate. Generally, this research investigates the main characteristics and parameters of the free hydraulic jump such as; the sequent depth, the relative hydraulic jump length and the relative distance to the jump. In the present research paper, these characteristics will be tested in rectangular channel downstream (DS) the vertical gate. The experimental program was conducted on a re-circulating flume with 2.5m long, 9cm wide and 30cm deep; with discharges range from 3 to 230 LPM. Statistical equation was developed to correlate the length, sequent depth ratio and distance of jump with the other independent parameters. Finally, clear matching of results from the length of jump was obtained.

Keywords: Open Channel, Hydraulic Jump, Energy Dissipation, Sloping Channel, Sluice Gate, water structure.

1. INTRODUCTION

Downstream of many types of hydraulic structures such as dams and barrages, sluice gates and draft tubes of hydraulic turbines, a considerable portion of the kinetic energy in supercritical flow must be dissipated to prevent scour and erosion. Chern [5] Study the effect of a corrugated bed on the hydraulic jump, a smoothed particle hydrodynamics (SPH) model is applied to investigate the characteristics of hydraulic jumps in various corrugated beds. It is found that the sinusoidal bed can dissipate more energy than other beds. As a result, corrugated beds can be used to enhance energy dissipation of hydraulic jump in the open channel. In general, the proposed SPH model is capable of simulating the effect of corrugated beds on hydraulic jump characteristics. Kordi [13] Study the transitional expanding hydraulic jump. The results indicate that the post depth Y_2 required to form an expanding jump is distinctly smaller than that for the corresponding classical jumps. The expanding jump length was 1.25 times the corresponding free jump length. A jump formed in a horizontal, wide rectangular channel with a smooth bed is often referred to as the classical hydraulic jump and has been studied extensively (Peterka [15]; Rajaratnam,[16], McCorquodale, [14], and Hager [8]). If Y_1 and V_1 are, respectively, the depth and mean velocity of the supercritical stream just upstream of the jump, with a Froude number of $Fr_1 = \frac{V_1}{\sqrt{g y_1}}$ where g is the acceleration due to gravity, the subcritical sequent depth Y_2 is given by the well-known Belanger equation Bélanger [4]

$$\frac{Y_2}{Y_1} = \frac{\sqrt{1 + 8F_{r1}^2} - 1}{2} \qquad (1)$$

Hager [9] analyzed the hydraulic jump theoretically and experimental means. Hughes [11] studied the hydraulic jump characteristics over several artificially roughened test beds in a horizontal rectangular flume with smooth side walls. Observations showed that boundary roughness reduces both the sequent depth and the length of a hydraulic jump, and that the observed reductions were related to both Froude number and the degree of roughness. The observed hydraulic jump characteristics were consistent with theory, and a

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proposed approximation for a theoretical hydraulic jump equation was found to compare favorably with the observed characteristics. Afzal [2] Investigated the stream-wise flow structure of a turbulent hydraulic jump over a rough bed rectangular channel. The hydraulic jump over a rough bed channel can be directly deduced from classical smooth bed hydraulic jump theory, provided the upstream Froude number is replaced by the effective upstream Froude number. Abdel-Azim [1] Study the effect of both positive and negative slopes on the hydraulic jump. The analysis of results indicated that both the inlet Froude number and the bottom slope have major effects while the inflow depth ratio has a minor effect on the depth ratio of the jump at the outlet. Chyan-Deng [6], and Smith [19] Study the hydraulic jump in an inclined rectangular chute contraction. They developed theoretical equations for the sequent-area and sequent-depth ratios for hydraulic jumps in the contraction considering the effects of contracting width and sloping bottom. Beirami [3] studied the hydraulic jumps in sloping channels and showed that the negative slope of the basin reduces the sequent depth ratio, while a positive slope increases the sequent depth ratio. Gandhi [7] Study the characteristics of supercritical flow in rectangular channel. This paper presents the results of a study undertaken to determine the effects of bed slop, on the hydraulic jump characteristic.

2. THEORY

The momentum equation applied between sections 1 and 2 is written as follows:

$$\rho Q v_1 + P_1 + W\sin\alpha = \rho Q v_2 + P_2 \tag{2}$$

Where ρ is the density, and Q is flow discharge. Figure 1 shows a hydraulic jump evolving in a channel with positive slope. The following hypotheses will be considered in sections 1 and 2: that the pressure is hydrostatic and the friction forces are negligible. The weight W of the jump and the pressure forces P1 and P2 can be expressed by applying the hydrostatics laws as

$$P_1 = m\omega\frac{Y_1^3\cos\theta}{3};\, P_2 = m\omega\frac{Y_2^3\cos\theta}{3};\, W = \omega V \tag{3}$$

where Y1 and Y2 are flow depths, v1 and v2 are average velocities, ϴ is the angle of inclination of the channel, V is the volume of water included between sections 1 and 2; m is the mass of water included between section 1 and 2; and 6 is the specific weight of the fluid. Geometrically, the volume V of the jump in a channel can be deduced from the quarter of a pyramid (Fig. 1).

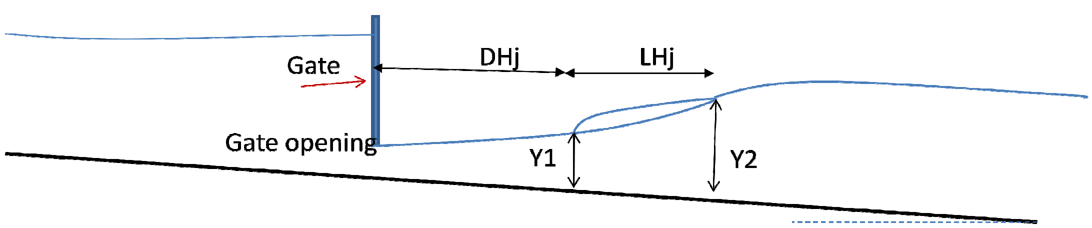

Fig. 1. Geometrical description of the volume of the jump.

The total energy losses between section 1 and section 2 were calculated by using the following formula (Rajaratnam [16]):

$$E_L = \frac{H3}{Hc} - \left(\frac{1+S_t}{2}\right)\times\left(\sqrt{\left(1+8(Fr1)^2\right)}-1\right) + \frac{(Fr1)^2}{2}\left(1-\frac{4}{(1+S_t)^2\times\left(\sqrt{\left(1+8(Fr1)^2\right)}-1\right)^2}\right) \tag{4}$$

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Where: St is the submergence ratio (yt-y2)/y2, yt is tail water depth, y2 is sequent water depth of the classical hydraulic jump.

In addition, the relative energy losses E_L/E_1 *100 was also calculated in order to check the efficiency of the design of new spillway stilling basin.

$$\Delta E = Y_1 + \frac{v_1^2}{2g} - Y_2 - \frac{v_2^2}{2g} \qquad (5)$$

3. THE EXPERIMENTAL WORK

The experiments were conducted in a recirculation self contained tilting glass sided flume in the Hydraulics Laboratory of Faculty of Engineering, Shoubra, Benha University. The flume is 2.5m long, 9cm wide and 30cm deep. A discharge control valve was used to regulate the flow rate. The bottom slope was adjusted using a screw jack located at the upstream end of the flume while at the downstream end; the flume was allowed to rotate freely about a hinged pivot. The slope was directly determined using a slope indicator. A downstream adjustable gate was used to regulate the tail-water surface elevation. The sidewalls along the entire length of the flume are made of glass with metal-frames, to allow visual investigation of the flow patterns. The horizontal bottom of the flume was made of steel and provided with a PVC pipe to drainage the water from the flume. The water entered the flume from an external water source, which was fed by an electric centrifugal pump. The water discharged the flume through two pumps with different capacities; 90, and 160lit/min. Figure 2 illustrates a complete and detailed flume description. A series of runs at different values of discharge were experimented and hydraulic jump was formed by operating the sluice gate and different discharge. For each run initial depth, sequent depth and length of hydraulic jump were measured. The experiments were carried out mainly by using five different gate opening, d_g of 1, 1.5, 2, 2.5, and 3cm. Five positive bottom slopes, S_o were used. Five slopes (0.0027, 0.004, 0.0054, 0.0081, and 0.011) were used in the experimental. Four different flow rates ranged from 90 to 150 LPM were used for each particular conduit height and bottom slope. The initial Froude number ranged from 1.2 to 4.6. The discharge was measured using a pre-calibrated orifice meter. Depth measurements were taken using a point gauge with an accuracy of ± 0.1 mm. For each run, the initial depth of jump, the flow rate and the depth of water just downstream the conduit outlet were measured. A total of 80 experiments were conducted and the primary details of these experiments are shown in Table 1.

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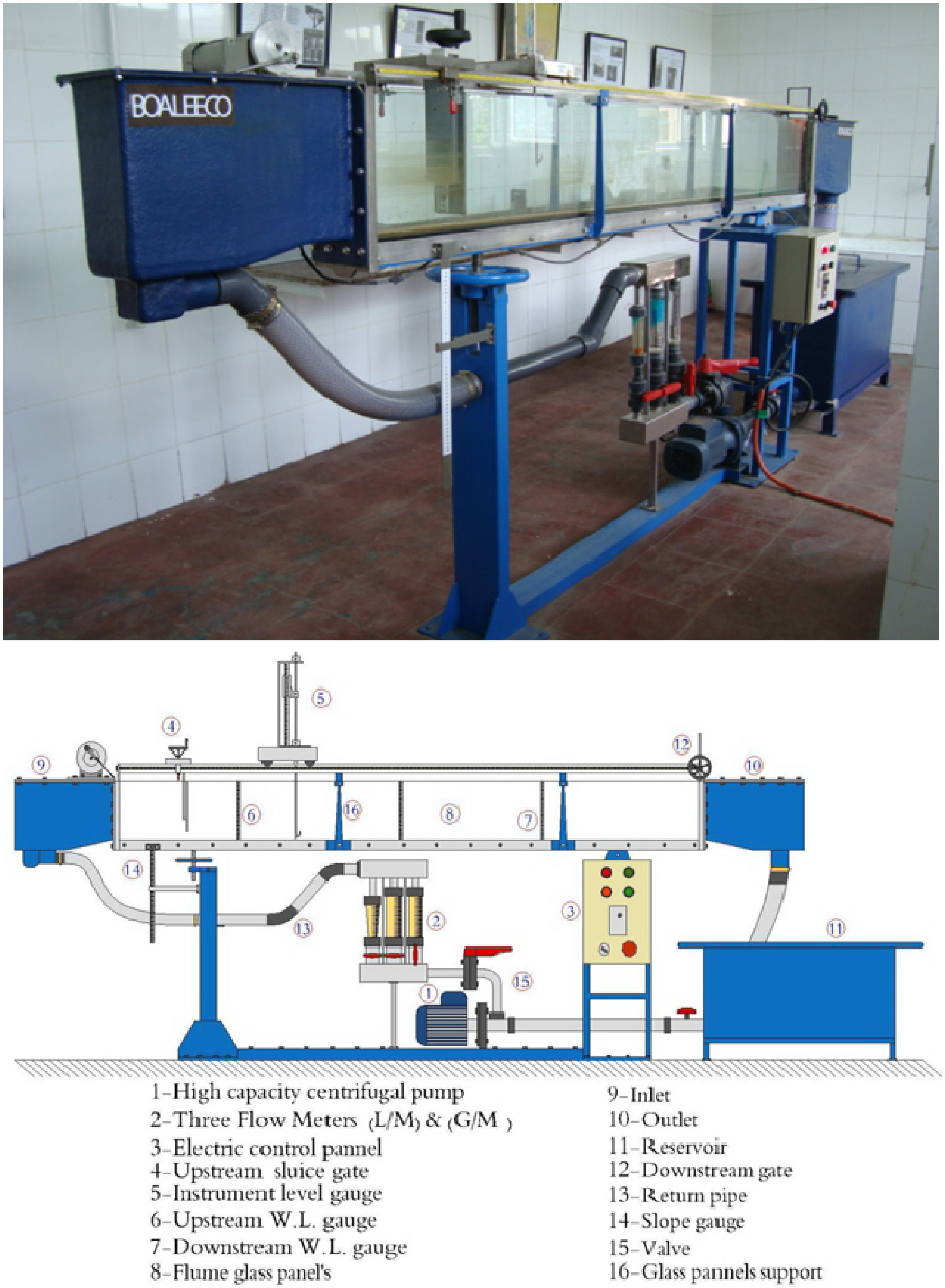

Fig. 2. Schematic Layout of Experimental Setup

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Table 1. Primary Details of Experiments

Run No.	Slope (S)	Gate opening (dg) cm	Q (LPM)
I1	0.0027, 0.004, 0.0054, 0.0081, and 0.011	1	90
I2	0.0027, 0.004, 0.0054, 0.0081, and 0.011	1	120
I3	0.0027, 0.004, 0.0054, 0.0081, and 0.011	1	135
I4	0.0027, 0.004, 0.0054, 0.0081, and 0.011	1	150
II1	0.0027, 0.004, 0.0054, 0.0081, and 0.011	1.5	90
II2	0.0027, 0.004, 0.0054, 0.0081, and 0.011	1.5	120
II3	0.0027, 0.004, 0.0054, 0.0081, and 0.011	1.5	135
II4	0.0027, 0.004, 0.0054, 0.0081, and 0.011	1.5	150
III1	0.0027, 0.004, 0.0054, 0.0081, and 0.011	2	90
III2	0.0027, 0.004, 0.0054, 0.0081, and 0.011	2	120
III3	0.0027, 0.004, 0.0054, 0.0081, and 0.011	2	135
III4	0.0027, 0.004, 0.0054, 0.0081, and 0.011	2	150
IV1	0.0027, 0.004, 0.0054, 0.0081, and 0.011	2.5	90
IV2	0.0027, 0.004, 0.0054, 0.0081, and 0.011	2.5	120
IV3	0.0027, 0.004, 0.0054, 0.0081, and 0.011	2.5	135
IV4	0.0027, 0.004, 0.0054, 0.0081, and 0.011	2.5	150
V1	0.0027, 0.004, 0.0054, 0.0081, and 0.011	3	90
V2	0.0027, 0.004, 0.0054, 0.0081, and 0.011	3	120
V3	0.0027, 0.004, 0.0054, 0.0081, and 0.011	3	135
V4	0.0027, 0.004, 0.0054, 0.0081, and 0.011	3	150

4. RESULTS AND DISCUSSION

The deviation of different hydraulic jump characteristics such as sequent depth ratio, relative height of the jump, distance to Hydraulic jump, and relative length of the jump, relative energy loss with approach Froude number and slope of the channel is given below.

4.1 THE SEQUENT DEPTH RATIO

The sequent depth ratio (Y_2/Y_1) was measured and plotted with respect to Froude number. Figure 3 shows the variation of sequent depth ratio with approach Froude number and slope of the channel. It is observed that sequent depth ratio increases with increase in approach Froude number and slope of the channel bed. It is obvious from the figure that approximately 95% and 99.7% of experimental data are laying within ± 10% of best fit curve. The increase of Froude number by 100% the relative sequent depth increases

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by 85%. At same Froude number the increase of channel slope causes increase of the relative depth. It is clear that the sequent depth ratio (Y_2/Y_1) increases with the increase of the slope at particular inlet Froude number. The lowest value is due to the minimum slope (0.0027) and the maximum value due to the maximum slope (0.011).

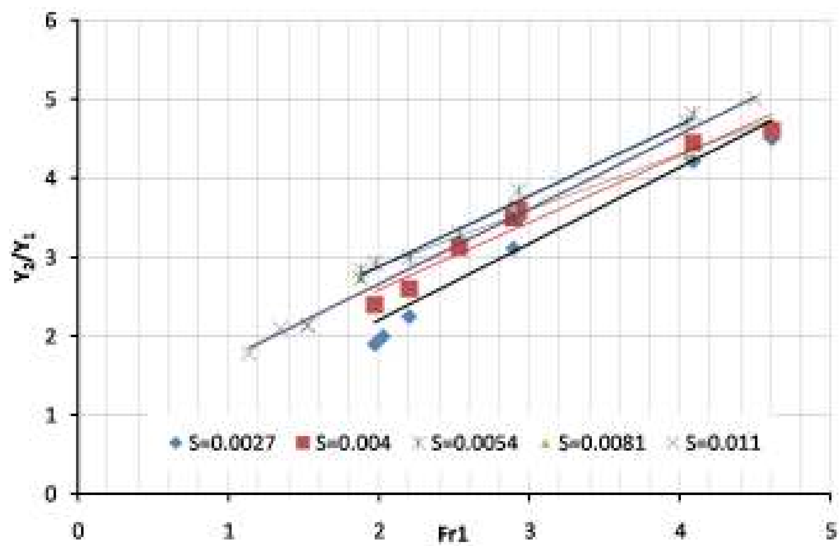

Fig. 3. Variation of Sequent Depth Ratio with approach Froude number and Channel Slope

Figures 4 presents the variations of Y_2/Y_1 with the bottom slope (0.0027, 0.004, 0.0054, 0.0081, and 0.011) for different inlet Froude number. It is clear that the depth ratio increases with the increase of the slope at particular inlet Froude number. The lowest value is due to the minimum slope (0.0027) and the maximum value is due to the maximum slope (0.011). This could be explained by the fact that, the weight component acts in the direction of the flow. This in turn results in a reduction of the depth ratio and the rate of reduction increase as the slope decreases. The multiple linear regressions were applied to predict a statistical equation that correlates sequent depth ratio with other independent parameters (F_{r1}, S_o, Q, dg, and Y_1) in the form;

$$\frac{Y_2}{Y_1} = -0.058 + 1.935 * F_{r1} + 0.1657 * F_{r1}^2 - \frac{0.0026}{S_o} \tag{6}$$

With correlation coefficient equals 0.95.

$$\frac{Y_2}{Y_1} = \exp\left(489.74Q + 9.38S_o - 71.8d_g + 1.57\right) \tag{7}$$

With correlation coefficient equals 0.90.

Where: Y_1 is the conjugate depth of flow U.S. the jump, Y_2 is the sequent depth of flow D.S. the jump, Q is the flow discharge, S_o is the slope of the channel, d_g is the gate opening, and F_{r1} is the Froude number at vena contracta.

Figure 5 shows the comparison of sequent depth ratio of present model equations 6, and 7 with the model equations developed by Rajaratnam [16] Husain [12] and Herbrand [10]. It is observed that most of the experimental data are lying between the present model and previous model. It proves that the present model gives high accuracy.

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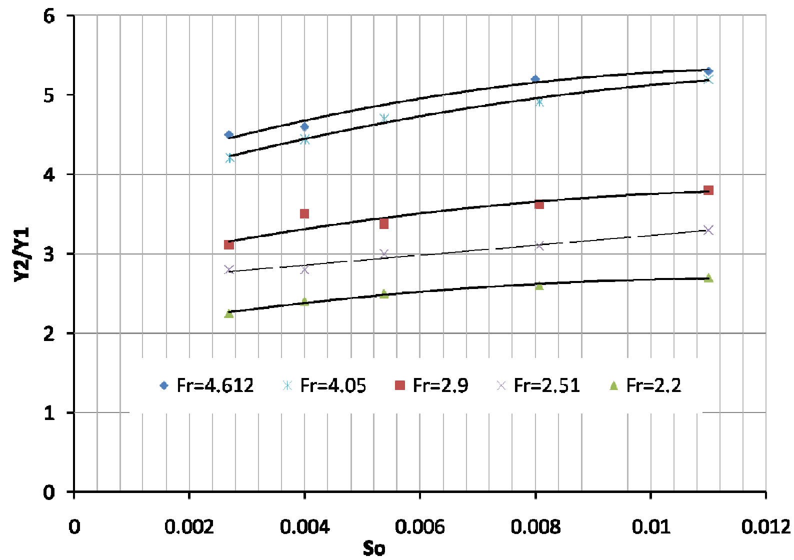

Fig. 4: Variations of Y2/Y1 with slope for different values of Froude number

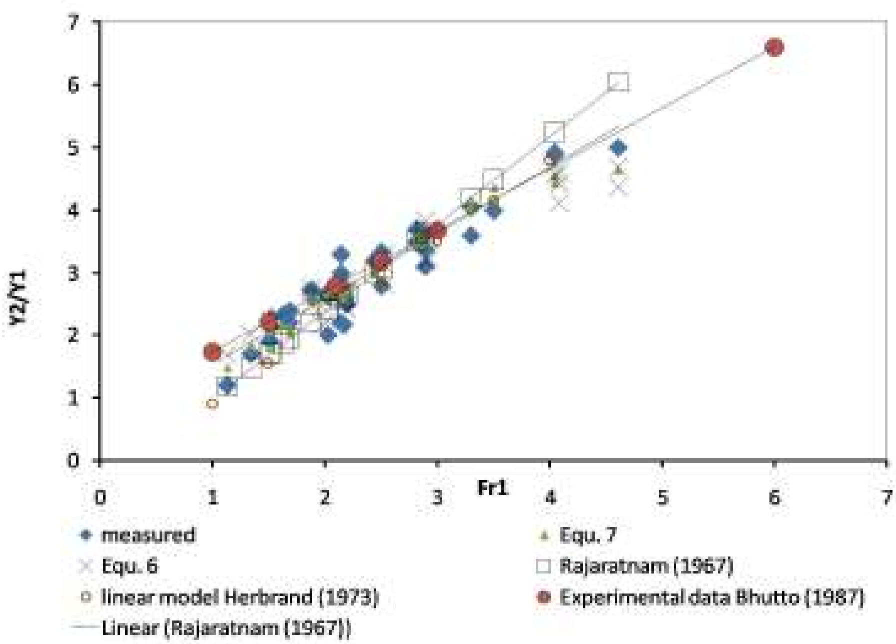

Fig. 5: The relation between the Froude number and the depth ratio

4.2 ENERGY DISSIPATION

Figure 6 shows the relation between the Froude number and relative energy loss (EL/E1) for different slope. It is observed that relative energy loss increases non-linear with increase in approach Froude number from 1.2 to 4.6 and slope of the channel bed. It is evident from the figure that approximately 98%, 97%, 93% and 92% of experimental data are laying within ± 8% of best fit curve.

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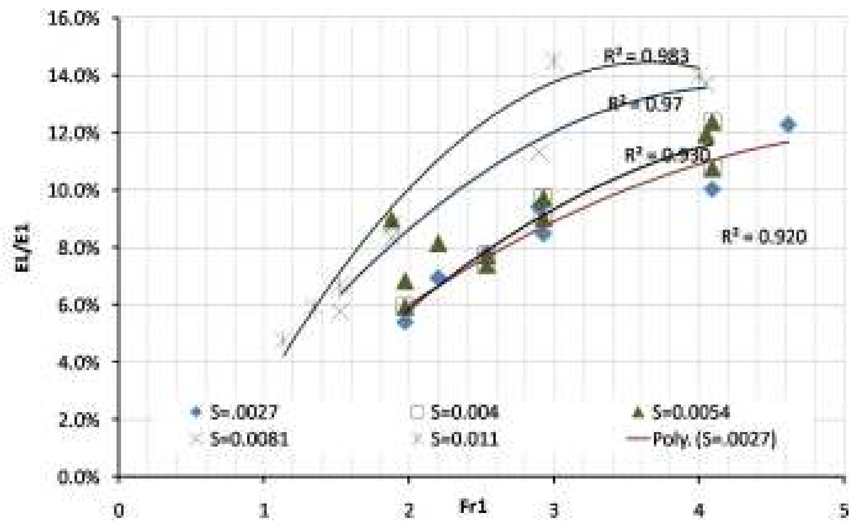

Fig. 6. Variation of Relative Energy Loss with approach Froude number and Channel Slope

4.3 LENGTH OF HYDRAULIC JUMP

Figure 7 illustrates the relation between hydraulic jump length (L_{Hj}) and the sequent depth (Y_2) for different slopes. It clears that the hydraulic jump length increase by 80% as increase of sequent depth by 100%. For same sequent length the hydraulic jump length increase by decreases the slope of the channel bed. Figure 8 shows the relation between the relative length of jump (L_{Hj}/Y_1) and the Froude number F_{r1} for different slope. It is observed that relative length of the jump increases with increase in approach Froude number and slope of the channel bed.

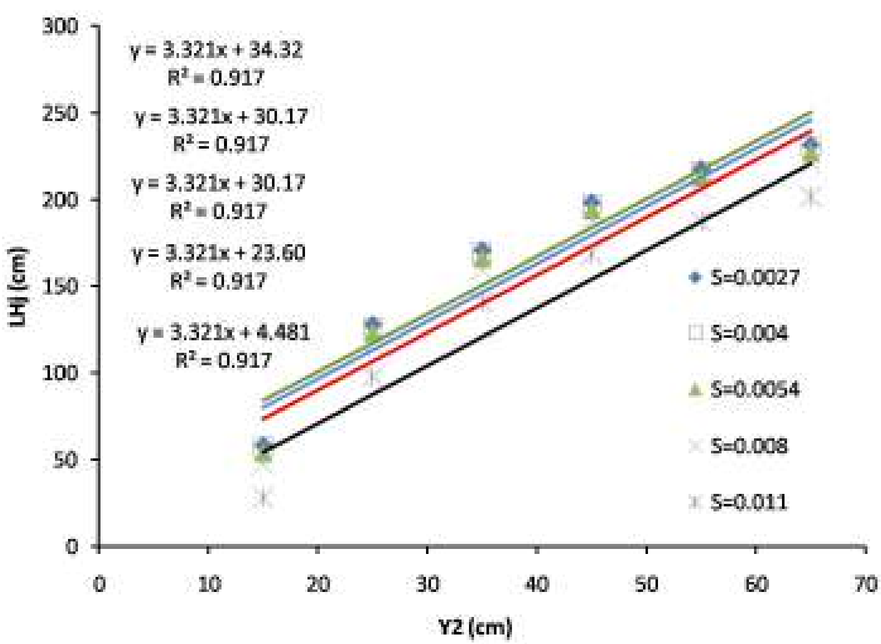

Fig. 7. The hydraulic jump length with approach sequent depth and Channel Slope

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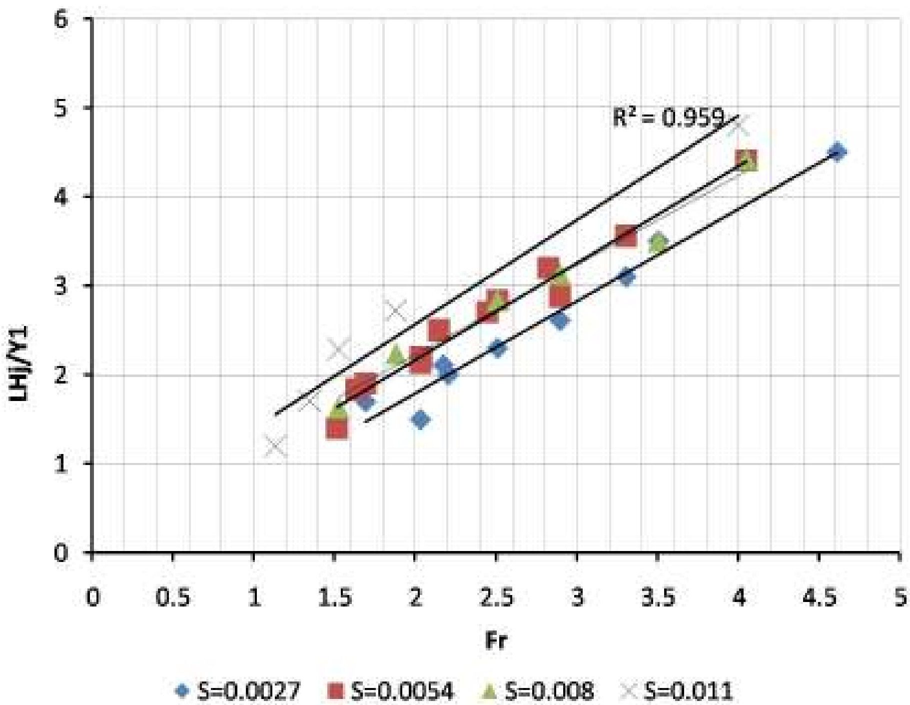

Fig. 8. The relative hydraulic jump length with approach Froude number and Channel Slope

In order to derive an equation with the corrected relative hydraulic jump length L_{Hj}/Y_1 as the dependent variable, the regression analysis tool of “Datafit ver. 9.0” was used to obtain the following equations:

$$L_{Hj}/Y_1 = 0.415 + 1.0315 * F_{r1} - 0.0017/S \qquad \mathbf{R^2 = 0.90} \qquad (8)$$

$$LHj/Y1 = 0.879(Y2/Y1) + 0.303 \qquad \mathbf{R^2 = 0.927} \qquad (9)$$

Figure 9 shows the comparison of relative height of the jump of present model equation 8 with the model equations developed by Rajaratnam [17], Husain Bhutto [12], and Herbrand [10]. It is observed that most of the experimental data of Husain [12] are lying between the present model and Herbrand [10] model.

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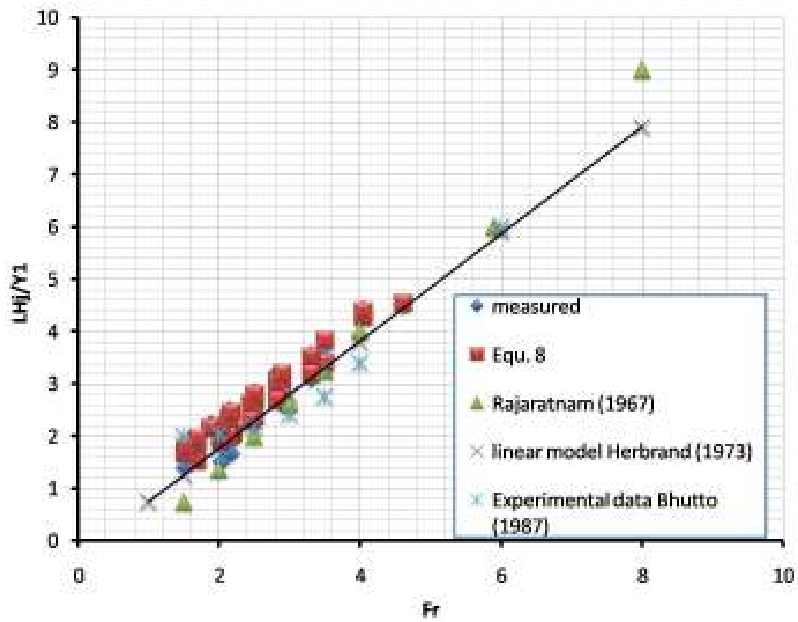

Fig. 9. The relative hydraulic jump length with approach Froude number and Channel Slope

4.4 DISTANCE TO HYDRAULIC JUMP

Figure 10 shows the relation between the relative distance of jump (D_{is}/Y_1) and the Froude number F_{r1} for different slope. It is observed that relative distance of the jump increases with increase in approach Froude number and slope of the channel bed. The multiple linear regressions were applied to predict a statistical equation that correlates D_{is}/Y_1 with other Froude number F_{r1} and different slope which represented by equation 9. Figure 11 shows the comparison between equation 10. Clearly good agreement was achieved.

$$\frac{D_{is}}{Y_1} = -52.3 + 183.13 * F_{r1} - 48.23 * F_{r1}^2 + 4.81 * F_{r1}^3 - \frac{1}{S_o} + \frac{0.0016}{{S_o}^2} \qquad \boldsymbol{R^2 = 0.92} \qquad (10)$$

This equation has a multi-linear correlation coefficient of 0.92. The computed values of D_{is}/Y_1 using the above equation are plotted against the corresponding experimental data as shown in Figure (9).

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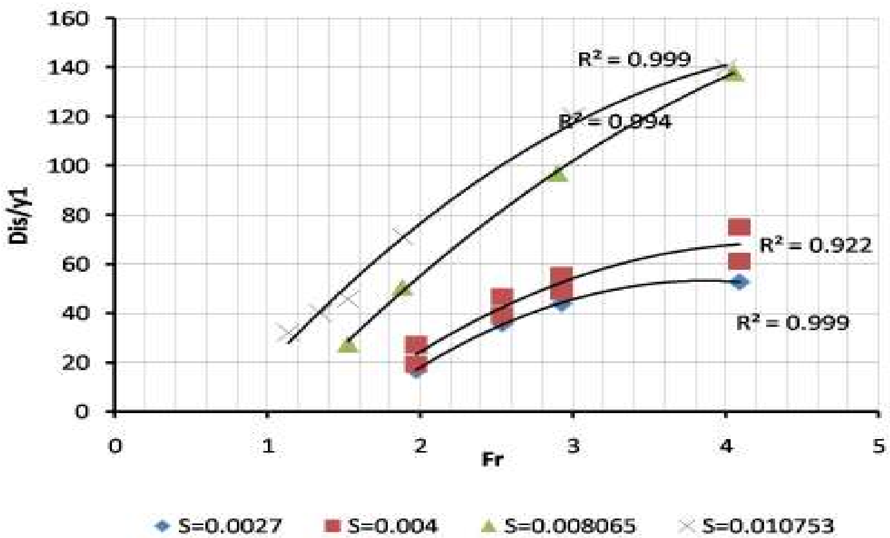

Fig. 10. Variation of Relative slap length with approach Froude number and Channel Slope

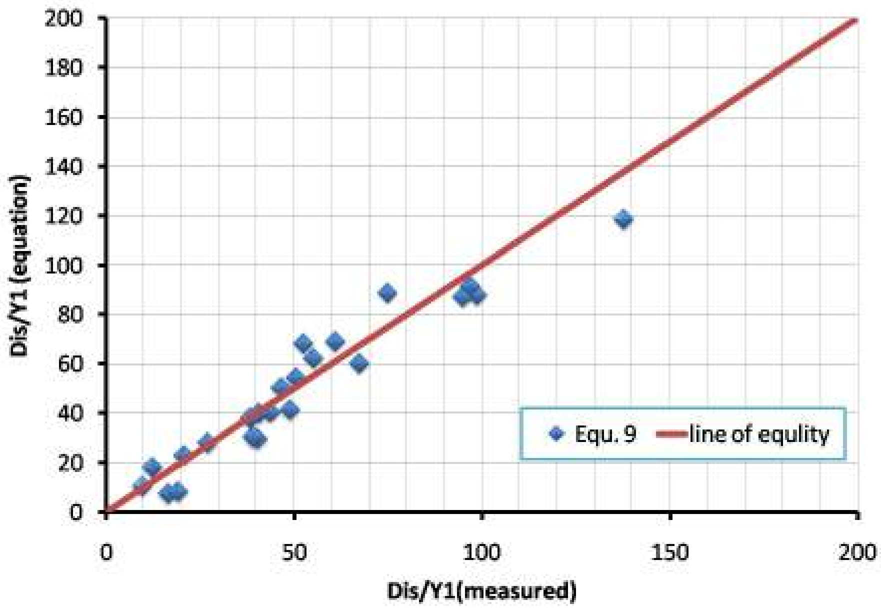

Fig. 11. Correlation between Dis/Y1 values, equation (10), and the corresponding observed onces

5. CONCLUSIONS

Hydraulic jumps produced in rectangular channel with different slopes and different Fraud numbers were analyzed. It was found that the bottom slope "S_o", and the inlet Froude number F_{r1}, have major effect on the variations of the jump outlet Characteristics. The sequent depth ratio increases with increase in approach Froude number and slope of the channel bed. The increase of Froude number by 100% the relative sequent depth increases by 85%. At same Froude number the increase of channel slope causes increase of the relative depth. The relative energy loss increases non-linear with increase Froude number and slope of the channel bed. The hydraulic jump length increases by 80% as increase of sequent depth by 100%. For same sequent length the hydraulic jump length increase by decreases the slope of the channel bed. The relative length of the jump increases with increase in approach Froude number and slope of the channel bed. The relative distance of the jump increases with increase in approach Froude number and slope of the channel bed. The prediction of the projected model was compared to the previously developed models. The predicted

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results agreed well with the experimental observations as well as with those of the before developed models using the same technique. The developed experimental computational models are applicable between Froude numbers from 1.2 to 4.6.

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List of symbols

Fr_1	Inflow Froude number [–]
g	Acceleration due to gravity [$m.s^{-2}$]
Y_1	Upstream sequent depth [m]
Y_2	Downstream sequent depth [m]
S	Channel slope [–]
L_{Hj}	Length of jump [m]
Ls	Length of slap [m]
Q	Flow discharge [$m^3.s^{-1}$]

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