Charge Radii of B and D mesons in a Quark Model with two loop static potential

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June 12, 2015

Abstract

We study the effects of two loop static potential on the charge radii of heavy light flavoured mesons in an Improved QCD potential model.The estimated value of charge radii of heavy-light mesons in V-scheme is then compared with other available data and found to be within the limit of other result.

Keywords: Two loop static potential, Dalgarno method and charge radii.
PACS Nos. 12.39.-x ; 12.39.Jh ; 12.39.Pn

1 Introduction

The study of hadron wavefunctions and their phenomenology is one of the important topics in QCD [1] and one of the reliable method of studying the physical properties of low energy QCD is the lattice formulation of gauge theory. But lattice QCD is limited to the Euclidean formulation of QCD and cannot be applied in Minkowski space time to simulate high energy reactions in which the particles are inherently moving near the light cone. It is also difficult to understand from simulations the important QCD mechanisms that lead to colour confinement.The only other way to proceed in the non perturbative regime of QCD is by inventing and using phenomenological models that capture the most important features of strong QCD. Since the classical work of De Rujula et al [2], such studies in non-relativistic potential approach have been made in several models [3, 4, 5, 6, 7, 8, 9, 10] with considerable success. Inspired by the success of this approach, the present authors have been pursuing a specific QCD Inspired Quark model [11, 12, 13, 14, 15] which has been developed in various stages : in earlier version [11, 12, 13, 14] significant confinement effects could not be introduced due to perturbative constraints, while in the later version [15] it could be incorporated.

The structure of the mesons can be learned by probing them with virtual photons in scattering experiment. The form factors of mesons not only provide a testing ground for QCD-inspired

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models, but also are important in many areas of particle and nuclear physics, including charge radii, parity-violating experiments, and many others. Present paper reports the results of charge radii of heavy light mesons within the improved version of the model, where the linear part of the Cornell Potential $V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + br + c$, is treated as perturbation and Coulombic part as parent to obtain an wavefunction by using Dalgarno method of perturbation[11, 13]. The corresponding results for the Isgur-Wise function and its slope and curvature have already been reported in [15].

In section 2 , we discuss the formalism , while in section 3 we summarise the results. Section 4 contains summary and conclusion .

2 Formalism

2.1 The Improved QCD Inspired Quark Model.

The essential features of the improved model has already been reported in ref [15]. With the linear part "$br + c$" of the Cornell potential as perturbation and coulombic part "$-\frac{4}{3}\frac{\alpha_s}{r}$" as parent, the wave function in this model is given by

$$\psi_{rel+conf}(r) = \frac{N'}{\sqrt{\pi a_0^3}}\left(C'(c) - \frac{1}{2}\mu b a_0 r^2\right)\left(\frac{r}{a_0}\right)^{-\epsilon} e^{-\frac{r}{a_0}} \tag{1}$$

where

$$N' = \frac{2^{\frac{1}{2}}}{\left[2^{2\epsilon}\left(\Gamma(3-2\epsilon)\left\{C'(c)\right\}^2 - \frac{1}{4}\mu b a_0^3 \Gamma(5-2\epsilon) C'(c) + \frac{1}{64}\mu^2 b^2 a_0^6 \Gamma(7-2\epsilon)\right)\right]^{\frac{1}{2}}} \tag{2}$$

is the normalisation constant; ϵ and a_0 are defined by

$$\epsilon = 1 - \sqrt{1 - \frac{4}{3}\alpha_S} \tag{3}$$

and

$$a_0 = \left(\frac{4}{3\mu\alpha_{\overline{MS}}}\right)^{-1} \tag{4}$$

respectively, with

$$C'(c) = 1 + cA_0\sqrt{\pi a_0^3} \tag{5}$$

A_0 being the undetermined factor appearing in the series solution of the Schrödinger equation and μ being the reduced mass.

From equation (5), if A_0 and/or c is set equal to zero, the effect of c disappears in the formalism as was the case for [11, 12, 13, 14, 16]. From equations (1) and (2), it is also evident that in the limit $b \to 0$, the effect of $C'(c)$ and hence of c completely disappears in the wave function.

2.2 Two loop static potential in V scheme

Traditionally, the strength of the quark gluon interaction is characterised by the coupling constant $\alpha_{\overline{MS}}(q^2)$ defined by convention in a particular dimensional regularisation scheme such as the $\overline{MS}$ regularisation scheme. V-scheme is a physically based alternative to this usual scheme. Indeed in lattice calculation,α_V is regarded as the better expansion parameter. From the points of convergence and scale ambiguities as well, V-scheme is advocated in recent literature. The relationship of $\alpha_V(\frac{1}{r^2})$ to the conventional coupling constant in $\overline{MS}$ scheme at the two loop level has been reported by Peter[17, 18]. Later, Schrö der has improved the work where the error in the gluonic sector has been corrected [19, 20].

Thus, the V-scheme (V denoting potential) [17, 18, 19, 20] defines the QCD coupling constant in terms of a potential, taking into account the higher order effects of QCD which are then expressed as a power series in the coupling constant $\alpha_{\overline{MS}}$ in $\overline{MS}$ scheme. In the V scheme α_V represents the effective coupling constant which incorporates the entire radiative corrections into its definition. For completeness, we write the relationship between α_V and $\alpha_{\overline{MS}}$ at the NNLO level which is given by

$$\alpha_V\left(\frac{1}{r^2}\right) = \alpha_{\overline{MS}}\left(1+\frac{\alpha_{\overline{MS}}}{4\pi}\left(a_1+\beta_0 L\right)+\right.$$
$$\left.\frac{\alpha^2_{\overline{MS}}}{16\pi^2}\left[a_2+\beta_0^2\left(L^2+\frac{\pi^2}{3}\right)+\left(\beta_1+2\beta_0 a_1\right)L\right)+O\left(\alpha^3_{\overline{MS}}\right)\right] \tag{6}$$

where

$$L = 2\ln\left(\tilde{\mu} r \exp\gamma\right), \tag{7}$$

$$\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f \tag{8}$$

and

$$\beta_1 = \frac{34}{3}C_A^2 - 4C_F T_F n_f - \frac{20}{3}C_A T_F n_f \tag{9}$$

are the first two terms of the renormalisation group beta function , $\tilde{\mu}$ being the scale parameter. The one-loop and two-loop constants with gauge independent coefficients are given by

$$a_1 = \frac{31}{9}C_A - \frac{20}{9}T_F n_f, \tag{10}$$

and

$$a_2 = \left(\frac{4343}{162}+4\pi^2-\frac{\pi^4}{4}+\frac{22}{3}\varrho\left(3\right)\right)C_A^2-\left(\frac{55}{3}-16\varrho\left(3\right)\right)C_F T_F n_f$$
$$+\frac{400}{81}T_F^2 n_f^2-\left(\frac{1798}{81}+\frac{56}{3}\varrho\left(3\right)\right)C_A T_F n_f \tag{11}$$

2.3 Form Factor and Charge Radii.

The elastic charge form factor for a charged system of point quarks has the form [21, 22]

$$e\left\{F\left(Q^2\right)\right\} = \sum_{i=1}^{2} \frac{e_i}{Q_i} \int_0^{\infty} r \mid \Psi(r) \mid^2 \sin(Q_i r)\, dr \tag{12}$$

where e_i is the charge of the i^{th} quark/antiquark and

$$Q_i = \frac{\sum_{j \neq i} m_j Q}{\sum_{i=1}^{2} m_i}. \tag{13}$$

Q_i describes how the virtuality Q is shared between the quark antiquark pair of the meson. For a pair of same flavour (light-light or heavy-heavy), it is shared equally ($Q_i = \frac{Q}{2}$), while for heavy-light, the entire virtuality is carried by the light quark. Using equation (1) in (12) and integrating over r

$$\begin{aligned} e\left\{F\left(Q^2\right)\right\} = \frac{N'^2}{a_0 2^{-2\epsilon}} \sum_{i=1}^{2} \frac{e_i}{Q_i} \Bigg[& C'^2 \Gamma(2-2\epsilon) \sin\left[(2-2\epsilon)\theta_i\right] \left(1 + \left(\frac{1}{4} a_0^2 Q_i^2\right)\right)^{(\epsilon-1)} - \\ & \frac{1}{4} \mu b a_0^3 C' \Gamma(4-2\epsilon) \sin\left[(4-2\epsilon)\theta_i\right] \left(1 + \left(\frac{1}{4} a_0^2 Q_i^2\right)\right)^{(\epsilon-2)} + \\ & \frac{1}{64} \mu^2 b^2 a_0^6 \Gamma(6-2\epsilon) \sin\left[(6-2\epsilon)\theta_i\right] \left(1 + \left(\frac{1}{4} a_0^2 Q_i^2\right)\right)^{(\epsilon-3)} \Bigg] \end{aligned} \tag{14}$$

where

$$\theta_i = \sin^{-1} \frac{Q_i}{\left(\frac{4}{a_0^2} + Q_i^2\right)^{\frac{1}{2}}}. \tag{15}$$

In evaluating (14) we use the approximation in (15) to be

$$\sin^{-1} x \approx x + \frac{x^3}{6} + \frac{3x^5}{40} \tag{16}$$

with

$$x = \frac{Q_i}{\left(\frac{4}{a_0^2} + Q_i^2\right)^{\frac{1}{2}}}. \tag{17}$$

We also use

$$\sin y = y - \frac{y^3}{3!} + \frac{y^5}{5!} \tag{18}$$

with

$$y = (2-2\epsilon)\theta_i. \tag{19}$$

As a result of these improved approximations, the expression for $e\left\{F\left(Q^2\right)\right\}$ in equation (14) becomes

$$e\left\{F\left(Q^2\right)\right\} = \frac{N_1^2}{a_0 2^{-2\epsilon}} \sum_{i=1}^{2} e_i \left[\left(1 + \frac{1}{4} a_0^2 Q_i^2\right)^{(\epsilon-1)} \left((2-2\epsilon) X_i - \frac{1}{6}(2-2\epsilon)^3 Q_i^2 X_i^3 + \right.\right.$$

$$\frac{1}{120}\left(2-2\epsilon\right)^5 Q_i^4 X_i^5\Big) C'^2\Gamma\left(2-2\epsilon\right) - \frac{1}{4}\mu b a_0^3 \left(1+\frac{1}{4}a_0^2 Q_i^2\right)^{(\epsilon-2)} \Big(\left(4-2\epsilon\right) X_i - \frac{1}{6}\left(4-2\epsilon\right)^3 Q_i^2 X_i^3 +$$
$$\frac{1}{120}\left(4-2\epsilon\right)^5 Q_i^4 X_i^5\Big) C'\Gamma\left(4-2\epsilon\right) + \frac{1}{64}\mu^2 b^2 a_0^6 \left(1+\frac{1}{4}a_0^2 Q_i^2\right)^{(\epsilon-3)} \Big(\left(6-2\epsilon\right) X_i -$$
$$\frac{1}{6}\left(6-2\epsilon\right)^3 Q_i^2 X_i^3 + \frac{1}{120}\left(6-2\epsilon\right)^5 Q_i^4 X_i^5\Big)\Gamma\left(6-2\epsilon\right)\Big] \quad (20)$$

where

$$X_i = \frac{3Q_i^4}{40}\left(\frac{4}{a_0^2}+Q_i^2\right)^{-\frac{5}{2}} + \frac{Q_i^2}{6}\left(\frac{4}{a_0^2}+Q_i^2\right)^{-\frac{3}{2}} + \left(\frac{4}{a_0^2}+Q_i^2\right)^{-\frac{1}{2}}, \quad (21)$$

N_1 being the normalization constant,

$$N_1 = 2^{\frac{1}{2}}\Big(2^{2\epsilon}\Big(\left(2-2\epsilon\right) C'^2\Gamma\left(2-2\epsilon\right) - \frac{1}{4}\mu b a_0^3\left(4-2\epsilon\right) C'\Gamma\left(4-2\epsilon\right) +$$
$$\frac{1}{64}\mu^2 b^2 a_0^6\left(6-2\epsilon\right)\Gamma\left(6-2\epsilon\right)\Big)\Big)^{-\frac{1}{2}}. \quad (22)$$

The average charge radii square for the mesons are obtained from the relation

$$\left\langle r^2\right\rangle = -6\frac{dF(Q^2)}{dQ^2}\,|_{Q^2=0}\,. \quad (23)$$

Using (20) in (23), one obtains for the mesons having quark masses m_i and m_j, and charges e_i and e_j respectively ,

$$\left\langle r^2\right\rangle = a_0^2\left[e_i\left(1+\frac{m_i}{m_j}\right)^{-2} + e_j\left(1+\frac{m_j}{m_i}\right)^{-2}\right]\Big(C'^2\Gamma\left(2-2\epsilon\right)\Big(\frac{(2-\epsilon)}{2}+\frac{(2-\epsilon)^3}{4}+$$
$$\frac{3\left(2-\epsilon\right)\left(\epsilon-1\right)}{2}\Big) - \frac{1}{4}\mu b a_0^3 C'\Gamma\left(4-2\epsilon\right)\left(\frac{(4-2\epsilon)}{2}+\frac{(4-2\epsilon)^3}{4}+\frac{3\left(4-2\epsilon\right)\left(\epsilon-2\right)}{8}\right)$$
$$+\frac{1}{64}\mu^2 b^2 a_0^6\Gamma\left(6-2\epsilon\right)\left(\frac{(6-2\epsilon)}{2}+\frac{(6-2\epsilon)^3}{4}+\frac{3\left(6-2\epsilon\right)\left(\epsilon-3\right)}{128}\right)\Big)\Big(\left(2-2\epsilon\right) C'^2\Gamma\left(2-2\epsilon\right) -$$
$$\frac{1}{4}\mu b a_0^3\left(4-2\epsilon\right) C'\Gamma\left(4-2\epsilon\right) + \frac{1}{64}\mu^2 b^2 a_0^6\left(6-2\epsilon\right)\Gamma\left(6-2\epsilon\right)\Big)^{-1}. \quad (24)$$

3 Result and Discussion

3.1 Relationship between $\alpha_{\overline{MS}}$ and α_V

Taking number of quark flavour $n_f = 4$ and $n_f = 5$ with the corresponding value of $\alpha_{\overline{MS}}$ at the scale of the c- and b- quark mass, $\alpha_{\overline{MS}}(m_c) = 0.39$ and $\alpha_{\overline{MS}}(m_b) = 0.22$ respectively [23], the value of $\alpha_V(\frac{1}{r^2})$ for three choices of $\widetilde{\mu}$ are calculated[14] and shown in Table 1.

It shows that the two loop potential invariably scales up the effective coupling constant from $\alpha_{\overline{MS}}$ to α_V. The increase in percentage is also shown in Table 1. It is much more for c-quark than for b-quark. The values of the running coupling constant for B and D mesons are $\alpha_{\overline{MS}} = 0.22$ and $\alpha_{\overline{MS}} = 0.39$ respectively [23]. The two loop calculations raise these values for B and D mesons

Table 1: Values of $\alpha_V(\frac{1}{r^2})$ for different choices of $\widetilde{\mu}$ and the percentage increase

Choices of $\widetilde{\mu}$	$\widetilde{\mu}=\frac{1}{r}$	%	$\widetilde{\mu}=\frac{1}{r}\exp\left(-\gamma_E-\frac{a_1}{2\beta_0}\right)$	%	$\widetilde{\mu}=\frac{1}{r^2}\exp\left(-\gamma_E\right)$	%
$\alpha_{\overline{MS}}(m_c)=0.39$, $n_f=4$	0.693	77.7	0.625	66.9	0.604	54.9
$\alpha_{\overline{MS}}(m_b)=0.22$ $n_f=5$	0.259	17.7	0.251	18.6	0.261	17.3

to $\alpha_V=0.261$ and $\alpha_V=0.625$ respectively as shown in Table 1, and are in good agreement with our results of [11] for D mesons. For B mesons however, the boost given by two loop results is insufficient.

3.2 Effect of the parameter c in the analysis

In the previous studies on this model [11, 12, 13, 14, 16], the undetermined factor appearing in the series solution of the Schrödinger equation A_0, (as occured in equation (5)), was set equal to zero and the effect of c disappeared altogether in the formalism. In the previous studies, a very small value of the confinement parameter b could only be accomodated due to the perturbative constaints. However, in the present analysis, the standard quarkonium spectroscopic result [24, 25] $b=0.183GeV^2$ can also be accomodated by a suitable choice of c (taking $A_0=1$), since the new perturbative constraints given by equations (12) and (13) of [15] do not prohibit large values of $C'(c)$. As mentioned in [15], since the reduced masses of mesons are about 1 GeV or less, a very large value of the parameter c occurring in the Hamiltonian given in equation (12) of [11] would most probably not be natural.

3.3 Charge radii

Using equation (24), we calculate the square of their charge radii $\langle r^2\rangle_F$ for finite heavy quark masses in fm^2 and compare our results with other results . We show our results for the mean square charge radii of the heavy pseudoscalar mesons in Table 2, taking $b=0.183GeV^2$ and $c=1GeV$ with $\alpha_V=0.625$ for the D mesons, and in Table 3 with $\alpha_V=0.6$ and $\alpha_V=0.261$ for the B mesons. Predictions for D mesons agree well with Hwang [9], but for B mesons, (with $\alpha_V=0.261$), the corresponding predictions overshoot by a factor of 2 - 3 . This anomalous feature can be overcome only if α_V for B mesons is higher, say $\alpha_V\sim 0.6$.

3.4 Conclusions

We have calculated the charge radii of heavy light flavoured mesons with two loop static potential within a QCD inspired quark model pursued by us in recent years. We have also improved our earlier calculations [11, 13, 14] as substantial confinement effects can now be accommodated in contrast to the previous version of the model. Relativistic effects have been estimated in the wave function at the origin by a procedure analogous to the hydrogen atom in QED. Before conclusion, we also make a few comments.

Table 2: Values of Mean Square Charge Radii of the Heavy-Light Pseudoscalar D Mesons in fm^2 for finite quark masses $\langle r^2 \rangle_F$ in the V scheme with $c = 1GeV$ for $b = 0.183GeV^2$ with $\alpha_V = 0.625$ and $A_0 = 1$ in comparison to the predictions of others.

	ϵ_V	D_S^+	D^0	D^+
$\langle r^2 \rangle_F$, this work,	0.592	0.077	-0.302	0.172
$\langle r^2 \rangle_F$[9]	-	0.124	-0.304	0.180
$\langle r^2 \rangle_{T,F}$ $(coulombic\ as\ parent)$[27]	-	0.0101	-0.0139	0.0119
$\langle r^2 \rangle_F$[28]	-	0.1260	-0.2343	0.1340
$\langle r^2 \rangle_F$[26]	-	0.1239	-0.3036	0.1849
$\langle r^2 \rangle_F$ $(nonrelativistic)$[29]	-	-	-0.210	0.114

Table 3: Values of Mean Square Charge Radii of the Heavy-Light Pseudoscalar B Mesons in fm^2 for finite quark masses $\langle r^2 \rangle_F$ in the V scheme with $c = 1GeV$ for $b = 0.183GeV^2$ and $A_0 = 1$ for both $\alpha_V = 0.261$ and $\alpha_V = 0.6$ in comparison to the predictions of others

	α_V	ϵ_V	B_C^+	B_S^0	B^0	B^+
$\langle r^2 \rangle_F$, this work	0.261	0.193	0.565	-5.401	-13.651	27.489
$\langle r^2 \rangle_F$, this work	0.6	0.552	0.018	-0.073	-0.185	0.373
$\langle r^2 \rangle_F$[9]	-	-	0.0433	-0.119	-0.187	0.378
$\langle r^2 \rangle_F$[28]	-	-	-	-	-1.464	-
$\langle r^2 \rangle_F$[26]	-	-	-	-	-0.1866	-
$\langle r^2 \rangle_{T,F}$ $(coulombic\ as\ parent)$[27]	-	-	0.0106	-0.0250	-0.0301	0.0605

The approach of the earlier version ($b \sim 0$) could be justified when the two constituent quarks are very heavy and their motion is restricted to the small area around the origin, such as in the case of charmonium or bottonium. For significant confinement effect ($b > 0$), no such restriction exists and is presumably applicable to the heavy-light mesons as well, as we have done in the present paper. The approach can be further tested for the excited heavy-light mesons and for their mass differences.

As we are considering only pseudo scalar mesons, the spin-spin interactions in equation (11) of [11] which give mass splitting between pseudoscalar and vector mesons is neglected. By redefining the c-term, this can be included.

In the present analysis, our results agree with current available data as well as with those of Hwang[9] for a value of the coupling constant at around α=0.6 to 0.65 instead of the running coupling constants of $\alpha_{\overline{MS}} = 0.39$ for D mesons and $\alpha_{\overline{MS}} = 0.22$ for B mesons. Such higher value of strong coupling constant can be generated in the V-scheme with $O\left(\alpha_S^3\right)$ terms for D mesons but falls short for B mesons. As noted earlier [14], this presumably indicates large flavour dependent higher order effects of $O\left(\alpha_S^3\right)$ for B mesons. Moreover, this article gives improved values of heavy-light quark mesons, which could be valuable for future theoretical research.

Although the present analysis is an improvement of the earlier work of [11, 13, 14] in the sense that significant confinement as well as higher order effects in α_S through V-scheme have been incorporated, it still falls short in explaining correctly the properties of mesons with b-quarks.

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