Abstract—In this paper,by using the matrix optics, the transmission of quratz crystal berifringent filter with any direction of incidence angle,azmuth,mechanical rotation angle was derived .Through theoretical simulation obtained the laws of center wavelength and free spectral range of quratz srystal berifringent filter changing with the mechanical-angle.The results show that rotated around the z-axis of the filter mostly influence of the features of the filter and the transmission spectrum of the designed filter is tested and investigated by using Ultra-6600 spectrophotometer. The experimental results are in good agreement with theoretical results and it is helpful for manufacturing, adjusting and correcet design of the quartz birefringence filter.

*Index Terms*—optical devices,crystal optics, mechanical-angle, quratz crystal birefringent filters

# 1. Introduction

Briefringence filter is one of the important passive components, including two classic types:Lyot filter[1,2] and Solc filter[3,4],which is widely used in the field of spectral images [5,6],lasertuning[7,8],retomesening[9],astronomy[10,11],dense wavelength division multiplexing[12]. For a optical ,working in fixed transmission wavelength spectra is very important.For example,in the sunlight observation instrument application, output central wavelength is sensitive to incident angle [13-15] ,in the birefringent filter system ,mechanical stress is inevitable,due to the mechanical stress ,the filter will produce a tiny rotation angle ,thus the accuracy of birefringent filter system decreased,therefore study the function of tiny rotation mechanical-angle and the transmission of the filter is useful for manufacturing, designing and adjusting quartz birefringence filter.

In this paper, the quartz crystal wave plate rotated around the optical axis (x-axis) or y-axis or z-axis, the spectral output of the filter is studied by the way of numerical simulation and experiment researchs in detail, through numerical analysis and experiments research, the change of the center wavelength and the free spectral range of the filter are investigated by using Ultra-6600 series spectrophotometer in detail, and the results of theory are agreed to the results of experiment very well.

The structure of this paper is that, firstly theoretically analyzed the principle of quartz filter. Secondly, the additional phase of the filter under tiny mechanical-angle is discussed Thirdly,the spectral outputs of the filter with any direction of incidence angle,azmuth,mechanical rotation angle are studied through numerical simulation and experiment researchs and analysis the experimental results ,then make a conclusion in the end.

## 2. Numerical simulation and theory anlysis

# A. Basic principle of quartz filter

A typical type optical filter is usually composed of several birefringence polarization devices and several polarizing devices[16-17].Schematic diagram of the unipolar filter is shown in Fig1 consist of polaroids P1,P2 and the quartz plate L,the optical axis of the plate is parallel to its surface.

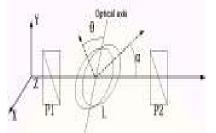


Fig.1 Schematic diagram of the unipolar birefringence filter

Where  $\alpha$  is the angle between crystal surface and the incident light, $\theta$  is the azimuth angle,L is the quartz plate.Base on the matrix optics,the Jones matrix of the quartz crystal given by:

$$M_{B} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

Where each element given by:

$$m_{11} = a_0 \left[ (1 - \frac{\sin \alpha_i}{q n_e^2}) \cos^2 \theta \exp(i \,\delta_e + \sin^2 \theta \exp(i \,\delta_0)) \right]$$
(2)

$$m_{12} = m_{21} = a_0 \left\{ \sqrt{1 - \frac{\sin^2 \alpha_i}{q n_e^2}} \sin \theta \cos \theta [\exp(i \,\delta_e) - \exp(i \,\delta_o)] \right\}$$
(3)

$$m_{22} = a_0 \left\{ \sin^2 \theta \exp(i \,\delta_e) + \left[ 1 + \frac{\sin^2 \alpha_i}{q n_e^2} \right] \cos^2 \theta \exp(i \,\delta_o) \right\}$$
(4)

Where 
$$a_0 = \left(1 - \frac{\cos^2 \alpha \sin^2 \alpha}{q q_e^2}\right)^{-1} \alpha_i$$
,  $q = 1 - \left(\frac{1}{n_o^2} - \frac{1}{n_e^2}\right) \sin^2 \alpha_i \cos^2 \theta$ ,  $\alpha_i$  is

the incidence angle  $(\alpha_i = \pi/2 - \alpha)$ ,  $n_e$ ,  $n_o$  are the refractive indices of the extraordinary and ordinary of the quartz plate. Based on matrix optics, the transmittance of the filter given by:

$$T = |m_{22}|^2 = 1 - A \sin^2 \frac{\delta'}{2}$$
 (5)

(1)

Where 
$$A = \frac{q^2 n_e^4 - q n_e^2 \sin^2 \alpha_i}{(q n_e^2 - \cos^2 \theta \sin^2 \alpha_i)^2} \sin^2 2\theta$$
,  $\delta'$  is the

total phase delay of the quartz crystal plate under tiny mechanical-angle  $\Delta \theta$ ,  $\delta$  is the phase delay of the quartz crystal plate without mechanical-angle,  $\Delta \delta$  is the additional phase under tiny mechanical-angle  $\Delta \theta$ , the function of the three given by :

$$\delta' = \delta + \Delta \delta \tag{6}$$

(9)

Where

$$\delta = \frac{2\pi d}{\lambda} \left[ \frac{\sqrt{n_e^2 n_o^2 - (n_e^2 \cos^2 \theta + n_o^2 \sin^2 \theta) \sin^2 \alpha_i}}{n_o} - \sqrt{n_o^2 - \sin^2 \alpha_i} \right]$$
(7)

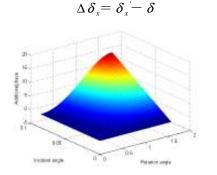
 $\lambda$  is the wavelength of incident light, *d* is the thickness of the quartz wafer, below the additional phase  $\Delta \delta$  of the filter under tiny mechanical-angle is discussed.

1). The quartz crystal rotated around the x-axis with tiny mechanical-angle

When the quartz crystal rotated around the x-axis of angle  $\Delta\theta$ , in the  $xy_1z_1$  oordinate system ,the direction of x-axis did not change,the incidence angle was still  $\alpha_i$ , rotation angle changed from  $\theta$  to  $\theta + \Delta\theta$ , the phase delay of the quartz crystal plate given by:

$$\delta_{x} = \frac{2\pi d}{\lambda} \left[ \frac{\sqrt{n_{e}^{2} n_{o}^{2} - [n_{e}^{2} \cos^{2}(\theta + \Delta \theta) + n_{o}^{2} \sin^{2}(\theta + \Delta \theta)] \sin^{2} \alpha}}{n_{o}} - \sqrt{n_{o}^{2} - \sin^{2} \alpha} \right]$$
(8)

and the additional phase give by:



**Fig.2** The additional phase  $\Delta S_x$  of the quartz plate as a function  $\alpha$  and

## $\Delta \theta$ when plate rotate around x-axis

2). The quartz crystal rotated around the y-axis with tiny mechanical-angle

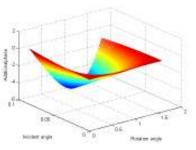
When the quartz crystal rotated around the y-axis of angle  $\Delta \theta$ , in the  $x_2 y z_2$  coordinate system, the direction of y-axis did not change, the incidence angle was still  $\alpha_i$  rotation angle changed from  $\theta$  to  $\theta + \Delta \theta$ , the phase delay of the quartz crystal plate given by:

$$\delta_{y}^{\prime} = \frac{2\pi d}{\lambda} \left\{ \frac{\sqrt{n_{e}^{2} n_{o}^{2} - n_{e}^{2} (\cos^{2} \theta \sin^{2} \alpha + n_{o}^{2} (\cos \alpha \sin \Delta \theta - \sin \alpha \sin \theta \cos \Delta \theta)^{2}}}{n_{o}} - (10) \right\}$$

 $\sqrt{n_o^2 - [(\cos\alpha\sin\Delta\theta - \sin\alpha\sin\theta\cos\Delta\theta)^2 + (\sin\alpha\cos\theta)^2]}$ 

and the additional phase give by:

$$\Delta \delta_{y} = \delta_{y} - \delta \tag{11}$$



**Fig.3** The additional phase  $\Delta \delta_v$  of the quartz plate as a function  $\alpha$  and

#### $\Delta \theta$ when plate rotate around y-axis

3). The quartz crystal rotated around the y-axis with tiny mechanical-angle

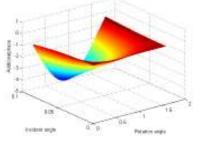
When the quartz crystal rotated around the y-axis of angle  $\Delta \theta$ , in the  $x_3 y_3 z$  coordinate system, the direction of y-axis did not change, the incidence angle was still  $\alpha_i$  rotation angle changed from  $\theta$  to  $\theta + \Delta \theta$ , the phase delay of the quartz crystal plate given by:

$$\delta_{z}^{i} = \frac{2\pi d}{\lambda} \left\{ \frac{\sqrt{n_{e}^{2} n_{o}^{2} - n_{e}^{2} (\cos \alpha \sin \Delta \theta + \sin \alpha \cos \theta \cos \Delta \theta)^{2} + n_{o}^{2} (\sin \alpha \sin \theta)^{2}}{n_{o}} - (12) \right\}$$

 $\sqrt{n_o^2 - [(\cos\alpha\sin\Delta\theta - \sin\alpha\sin\theta\cos\Delta\theta)^2 + (\sin\alpha\cos\theta)^2]}$ 

and the additional phase give by:

$$\Delta \delta_z = \delta_z - \delta \tag{13}$$



**Fig.4** The additional phase  $\Delta \delta_z$  of the quartz plate as a function  $\alpha$  and

 $\Delta \theta$  when plate rotate around z-axis.

Thus the transmittance of the emergent ligh given by:

$$T = 1 - A\sin^2\frac{\delta + \Delta\delta}{2} \tag{14}$$

From formula(14)we can see that a certain wavelength  $\lambda$  of incident light, corresponding to a certain phase delay  $\delta$ , we take a corresponding value  $\delta$ , can maximum value T, for different wavelength value  $\lambda$  of incident light, we need to change the  $\Delta\delta$ , in order to maximum value T, and the output of quartz plate changed.

### B. Numerical simulation analysis

Fig 3 show the theoretical transmission versus wavelength with different directions tiny mechanical angle with the wavelength range of 580~600nm. The thickness of quartz is 7mm the parameter of quartz used as follows:  $n_e$ ,  $n_o$  are 1.55335 and 1.54424 [11],  $\alpha_i$  is 4°,  $\theta$  is 45° rotation angle  $\Delta \theta$  is 3°, curve1, curve2, curve3, curve4 are the theoretical

transmission curves with the same rotation angle  $\Delta \theta = 3^{\circ}$  that rotated around the z-axis, rotated around the y-axis, rotated around the x-axis, and without rotation angle.

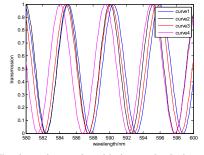
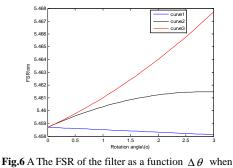


Fig.5 The simutation resules with tiny mechanical-angle at different axial directions

Form Figure 2 ,we can see that when the incidence angle  $\alpha = 4^{\circ}$ , the azimuth angle  $\theta = 45^{\circ}$  The quartz crystal rotated around the x-axis with tiny mechanical-angle  $\Delta \theta = 3^{\circ}$ , the central wavelength will change to shorter wavelength for about 0.1nm, the quartz crystal rotated around the y-axis with tiny mechanical-angle  $\Delta \theta = 3^{\circ}$ , the central wavelength will change to a shorter wavelength for about 0.3nm, the quartz crystal rotated around the z-axis with tiny mechanical-angle  $\Delta \theta = 3^{\circ}$ , the central wavelength for about 0.3nm, the quartz crystal rotated around the z-axis with tiny mechanical-angle  $\Delta \theta = 3^{\circ}$ , the central wavelength will change to a longer wavelength for about 0.9nm.

Figure 3 show the curve between the mechanical-angle and FSR of the filter. The thickness of quartz is 7mm,  $\alpha_i$  is 4°,  $\theta$  is 45°, rotation angle  $\Delta \theta = 0^\circ$ : +3°. From figure 3 we can see that the quartz crystal rotated around the x-axis, y-axis,z-axis ,the maximum offsets of the free spectral range(FSR) are -0.001nm,0.003nm,0.009nm,the deviations are -0.02%, 0.05% 0.16%, we can know that when  $\Delta \theta = 0^\circ$ : +3°, rotated around the z-axis mostly influence the FSR of the filter.



 $\alpha = 4^{\circ}, \theta = 45^{\circ}$ 

Figure 4 show the curve between the mechanical-angle and the center wavelength of the filter The thickness of quartz is 7mm,  $\alpha_i$  is 4°,  $\theta$  is 45°, rotation angle  $\Delta \theta = 0^\circ$ : +3°. From figure 4 we can see that the quartz crystal rotated around the x-axis, y-axis,z-axis the maximum offsets of the center wavelength are 0.1 nm,-0.3nm,-0.9nm,the deviations are 0.02%, -0.05%, -0.15%, we can know that when  $\Delta \theta = 0^\circ$ : +3°, rotated around the z-axis mostly influence the center wavelength of the filter. Therefore, by changing crystal obliquity angle can tune the center wavelength of the birefringent filter.

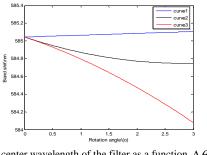


Fig. 7 The center wavelength of the filter as a function  $\Delta \theta$  when  $\alpha = 4^{\circ}$ ,  $\theta = 45^{\circ}$ 

# 3. Experimental results and discussion

In order to verify the correctness of the above theory, the experimental system is set up by using Ultra-6600 spectro photometer with 0.1nm sweeping step and 190nm to 900nm sweeping range, receive the transmission spectrum of quartz birefringence filter. The transmission spectrum measurement system framework shown in figure.5.

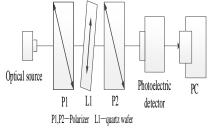
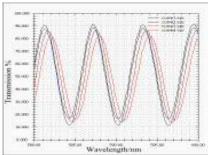
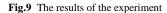


Fig.8 The structure of the mearsure system

The thickness of quartz wafer used in the test is 7mm, the incidence angle is about  $\alpha = 4^{\circ}$ , the azimuth angle is about  $\theta = 45^{\circ}$ , the transmission curve obtained of quartz birefringent filter is shown in Figure 6, the spectral range is 580~600nm, the spectral resolution is 0.1nm, curve1, curve2, curve3, curve4 are the transmission curves with the same rotation angle  $\Delta \theta = 3^{\circ}$  that rotated around the z-axis, rotated around the y-axis, rotated around the x-axis, and without rotation angle.





We can find the results of experiment are very similar to the results of Fig. 5 botained by numerical simulation. The experimental results show that : a). the central of quartz filter will change to a shorter wavelength for about 0.2nm,0.4nm,0.9nm with the same rotation angle of  $\Delta \theta = 3^{\circ}$ rotated around x-axis,y-axis,z-axis (theoretical values are 0.1 nm, 0.3nm, 1nm), the experiment results are in good agreement with the theoretical simulations, we believe that the presence of small differences are due to the thickness error of the measurement and the mechanical angle error. b). The maximum transmittance value of the filter is about 92%, we believe that this is due to the scattering of quartz wafer and the absorption of polarizer. c). The minimum transmittance value of the filter is about 10%, we believe that this is due to the extinction ratio of the polarizers is not high and the angle between the polarizer and the crystal is not strictly 45.

# 4. Conclusion

By using the matrix optics, the transmission of quartz birefringence filter with any direction of incidence angle,azmuth,mechanical rotation angle,through computer programming simulation.the transmission spectrum of quartz birefringence filter under tiny mechanical-angle is investigated in detail, the experimental results verify the correctness of the above theories, the results show that rotate around different axis the outputs of the quartz filter are different, the offset is related to the direction of rotation,and rotated around z-aixs mostly influence the the quartz filter features.It will has a certain application in stability output of optical filter when tiny mechanical-angle acts on filter .As a result ,by changing crystal obliquity angle can tune the center wavelength of the birefringent filter.The result may be useful for constructing high-performance birefrigent filter.

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